When will the Covid-19 pandemic peak?

Oliver Linton*
University of Cambridge, 31st March 2020

Abstract

We carry out some analysis of the daily data on number of new cases and number of new deaths by country as reported to the European CDC. We work with a quadratic time trend model applied to the log of new cases for each country. We use our model to predict when the peak of the epidemic will arise in terms of new cases or new deaths. We find that for the UK, this peak is mostly likely to occur within the next two weeks.

1 Purpose

We compare the progress of COVID-19 on countries worldwide with the hope of finding evidence of future turnaround of the upward trends. We will provide a weekly update.

2 Trend Modelling

We use daily data on new cases and fatalities downloaded from the website of the European Centre for disease protection and control. According to that website, the first case worldwide was recorded as December 31st 2019 (day 1). We have the daily number of (new) cases and the number of deaths upto March 31st, which is 92 days since day 1. We consider regressions of the form

$$\log(y_{it}+1) = m_i(t) + \varepsilon_{it},$$

where m_i is the trend in mean and y_{it} is either the number of new cases or the number of new deaths. We adopt a general to specific methodology. We first show the nonparametric (rolling window) local

*Thanks to Vasco Carvalho and Giancarlo Corsetti for comments. Cambridge INET and Faculty of Economics, Austin Robinson Building, Sidgwick Avenue, Cambridge, CB3 9DD. Email: obl20@cam.ac.uk.

linear kernel regression estimate for the number of cases (Figure 1) and number of deaths (Figure 2) in China, which is the country with the longest exposure, along with 95% pointwise confidence band.

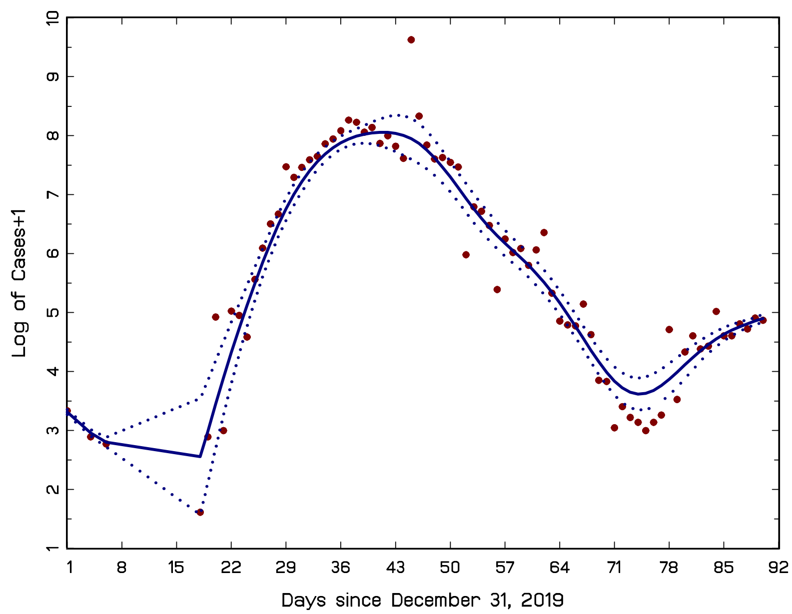

Figure 1

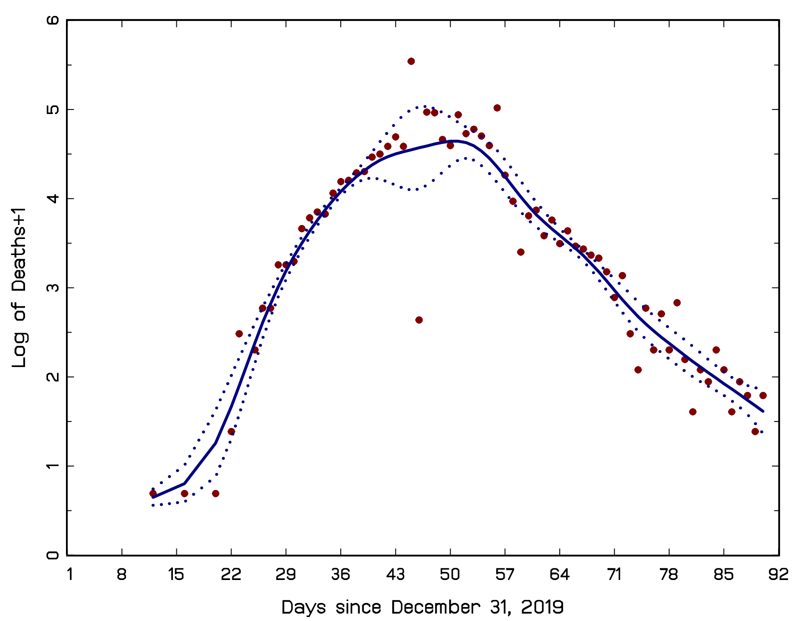

Figure 2

As one can see from the graphic, China has passed its (first) peak.

3 Quadratic Regression

We next consider a quadratic regression fit to country-wide count data $(y_{it}+1)/n_i$, where n_i is the population of country i, that is, we take

$$m(t) = \alpha + \beta t + \gamma t^2. \tag{1}$$

A quadratic is the simplest function that reflects the main features of the Chinese data, i.e., the possibility of a turning point; it can also be interpreted as a local approximation to a smooth trend function. Most media reports show log linear trends. Division by population only affects the constant term, but is done to aid comparability across countries. If time is rescaled to the unit interval, then the average trend is $\beta + \gamma$. A well defined maximum of m occurs provided $\gamma < 0$ and occurs at the time $t_{\max} = -\beta/2\gamma$, which results in the maximal value of cases per day of $m(t_{\max}) = \alpha - \beta^2/4\gamma$; finally, the value of t when no cases is achieved is the larger root of $m(t) = 0$. We explain in more detail in the appendix how we may estimate $t_{\max}$ and provide standard errors for it. Intuitively, we can identify the turning point from the curvature of the regression function when we approach such a turning point. We consider the countries with the largest number of cases (excluding China) and select those with at least twenty days of data. We then fit the quadratic regression on each country with the most recent twenty datapoints (using rescaled time) and report the results below in Tables 1 (cases) and Table 2 (fatalities).

The regressions generally have high R^2 but not 100%, and partly reflects data quality and country size. The first order serial correlation implies some short term predictability relative to the trend and this varies across countries; it would affect somewhat the standard errors although we have not adjusted for this. There is also some heteroskedasticity although this is generally decreasing over time. The contemporaneous correlation accross country residuals is quite variable with some large and positive and some large and negative, perhaps reflecting different lags in reporting and time zone effects. There are quite a few negative γ parameters in Table 1, which are indicative of future turning points, although mostly they are not statistically significant, yet. The α and β parameters are all strongly significant so we do not show the standard errors. There is quite a bit of heterogeneity across the parameters β, γ (with less so for α) consistent with different countries being at different stages of the cycle and taking different approaches to managing the epidemic.

The count data is subject to many errors, perhaps the main one is the undercount due to undertesting. The fatality count is perhaps more accurate although this is also subject to some errors. Some countries such as Italy have reach significantly negative γ already in Table 2.

Country	α	β	γ	se(γ)	R^2	$\rho(1)$	Cases	Population
USA	-14.8272	8.8794	-3.5710	0.6750	0.9805	0.4316	164620	327167434
Italy	-11.2541	5.1681	-3.3415	2.4136	0.3368	-0.2375	101739	60431283
Spain	-11.8084	6.1102	-3.0243	0.5796	0.9586	0.1878	85195	46723749
Germany	-13.2381	8.1689	-4.7669	1.1198	0.8859	-0.0162	61913	82927922
France	-12.1946	4.0336	-1.5408	0.5825	0.9371	-0.3750	44550	66987244
Iran	-11.1691	-0.9374	2.0803	0.5259	0.8476	0.5886	41495	81800269
United Kingdom	-14.1325	6.3599	-2.2846	0.9419	0.9384	-0.0804	22141	66488991
Switzerland	-11.5516	3.9131	-0.9929	4.1528	0.2886	-0.1975	15412	8516543
Belgium	-12.7230	5.6680	-1.9167	0.9384	0.9287	0.2608	11899	11422068
Netherlands	-12.6025	5.2972	-2.3348	0.4721	0.9697	-0.1018	11750	17231017
South Korea	-12.7054	-1.8896	1.6238	0.8448	0.1982	0.0719	9786	51635256
Austria	-12.3587	6.9790	-4.1126	0.6970	0.9345	-0.1459	9618	8847037
Canada	-14.9451	6.3431	-1.8670	1.1508	0.9252	-0.0048	7424	37058856
Portugal	-14.6757	10.9436	-6.0199	0.9930	0.9532	-0.1249	6408	10281762
Brazil	-17.3931	9.2999	-5.0694	1.0620	0.9292	0.0995	4579	209469333
Australia	-14.8738	8.1469	-4.4045	1.1563	0.8964	-0.5153	4557	24992369
Israel	-13.5901	4.7138	-0.6986	2.7176	0.6440	-0.4905	4473	8883800
Norway	-11.6807	3.2596	-1.4969	3.4156	0.1782	-0.2473	4226	5314336
Sweden	-11.3122	-0.9293	2.0702	1.0285	0.5913	0.0460	4028	10183175
Czech R	-13.3679	5.3720	-2.7453	1.2018	0.7966	-0.0280	3002	10625695
Ireland	-13.3296	7.1251	-3.5715	0.7896	0.9431	-0.3652	2910	4853506
Malaysia	-15.0622	8.4288	-5.8463	1.2592	0.8005	0.0586	2626	31528585
Denmark	-10.2713	-4.7081	5.0084	1.3550	0.4715	0.1114	2577	5797446
Chile	-15.5256	8.1085	-3.6641	0.9324	0.9487	-0.3526	2449	18729160
Phillipines	-16.4887	-0.7060	4.8161	4.3245	0.4780	-0.4575	2084	106651922
Poland	-15.6767	6.8052	-3.2385	0.6233	0.9639	-0.2612	2055	37978548
Luxembourg	-13.6879	15.0082	-10.2582	2.1933	0.8145	-0.3026	1988	607728
Ecuador	-17.3327	15.2528	-10.0678	1.4624	0.9190	-0.0887	1966	17084357
Japan	-14.4974	-3.5040	4.6870	1.7875	0.4744	0.3650	1953	126529100
Romania	-14.2848	2.5938	0.5899	1.1776	0.8624	0.1467	1952	19473936

Table 1. Counts data 20200311-20200331

Country	α	β	γ	se(γ)	R^2	$\rho(1)$	Deaths	Population
Italy	-13.0880	3.7533	-1.8475	0.5084	0.9199	-0.1897	11591	60431283
Spain	-15.8788	9.1566	-4.0993	1.4659	0.9064	-0.6880	7340	46723749
USA	-18.6605	5.3113	0.3673	2.7249	0.7861	-0.2665	3170	327167434
France	-16.5524	7.2991	-2.7412	1.0058	0.9435	-0.1389	3024	66987244
Iran	-14.2824	3.2173	-2.4122	0.3073	0.8846	0.6910	2757	81800269
UK	-18.1418	9.0303	-3.5979	1.8469	0.8755	-0.3544	1408	66488991
Netherlands	-17.0641	8.9043	-3.6879	1.3716	0.9216	-0.0496	864	17231017
Germany	-18.2654	3.2232	1.8432	2.4266	0.7928	-0.0896	583	82927922
Belgium	-16.6208	4.3853	0.8463	2.0007	0.8540	0.1284	513	11422068
Switzerland	-15.6690	3.2832	0.2841	1.6023	0.8078	-0.0371	295	8516543
South Korea	-16.2966	0.0486	0.5937	1.7008	0.1209	-0.1539	163	51635256
Brazil	-19.8251	4.1421	0.0198	0.9789	0.9383	0.4726	159	209469333
Sweden	-16.1106	1.4924	1.7644	1.8269	0.7412	-0.0926	146	10183175
Portugal	-16.5925	2.3066	1.6842	1.0085	0.9327	0.1111	140	10281762
Indonesia	-19.6225	3.5273	-0.6484	2.3310	0.5575	0.0807	122	267663435
Austria	-16.1627	1.6547	1.2983	2.3226	0.5892	-0.1624	108	8847037
Canada	-17.7783	3.1018	-0.3744	1.8122	0.6531	-0.2384	89	37058856
Phillipines	-18.0045	0.0747	1.9499	2.2614	0.4378	-0.6675	88	106651922
Denmark	-15.9886	3.0720	-0.2708	1.7681	0.6767	-0.3895	77	5797446
Ecuador	-16.8315	1.6463	0.8473	1.6833	0.6575	-0.3016	62	17084357
Japan	-17.1729	-1.8640	1.6350	1.2869	0.0920	-0.2437	56	126529100
Ireland	-14.9212	-2.8755	5.1181	1.2459	0.8108	-0.3447	54	4853506
Iraq	-17.5600	2.6834	-1.2740	1.7220	0.3533	0.0220	46	38433600
Romania	-16.8614	-0.5417	3.0295	1.1993	0.8150	0.2577	44	19473936
Greece	-16.0792	1.0382	0.7168	1.3349	0.6042	0.0127	43	10727668
Czech R	-16.1018	-1.2927	3.4435	1.6693	0.6496	-0.2325	42	10627165
Egypt	-18.4655	1.2273	0.6045	1.5027	0.5649	0.1200	40	98423595
Malaysia	-17.5954	1.9546	0.0822	1.5223	0.6018	0.0500	37	31528585
Morocco	-17.0787	-1.7572	3.2578	1.6943	0.5021	-0.2889	33	36029138
India	-20.7662	-0.7691	2.3356	1.4152	0.5714	-0.2086	32	1352617328

Table 2. Fatality data 20200311-20200331

4 Turning Point Estimation

We present the estimated turnaround time for selected countries (which have $\gamma < 0$) along with the 95% confidence interval in Table 3 and 4. Surprisingly, the USA is predicted to turnaround in around 5 days (plus or minus 6 days, so with high confidence before two weeks) in terms of case; however, it does not have a prediction yet in terms of deaths as the curve has some way to go to turnaround. The UK is predicted to turnaround in around 8 days (plus or minus 15 days) in cases and sooner in terms of deaths. The peak in cases should precede the peak in deaths but this is not imposed in our estimation, and data issues may lead to violations of this.

Country	Turnaround in Days	$\pm Days$
USA	5.1086	5.8022
Spain	0.2136	4.0493
France	6.4872	12.6321
UK	8.2299	15.1950
Switzerland	20.3814	255.1463
Belgium	10.0512	19.8106
Netherlands	2.8219	5.2990
Canada	14.6742	30.6006
Israel	49.8450	466.0467
Norway	1.8640	55.5232
Chile	2.2364	6.3769
Poland	1.0642	4.3841

Table 3. Predicted turnaround in number of cases in days from 20200331

Country	Turnaround in Days	$\pm Days$
Italy	0.3311	5.8765
Spain	2.4535	9.1131
France	6.9586	12.6018
UK	5.3536	16.0044
Netherlands	4.3520	10.8624
Indonesia	36.1237	332.2233
Canada	65.9957	736.2025
Denmark	98.1342	1412.7716
Iraq	1.1157	30.9216

Table 4. Predicted turnaround in number of deaths in days from 20200331

5 Appendix

Suppose that $m(t) = \alpha + \beta t + \gamma t^2$. A well defined peak of m occurs provided $\gamma < 0$ and occurs at the point $t_{\max} = \frac{\beta}{-2\gamma}$ with maximal value $m(t_{\max}) = \alpha + \beta t_{\max} + \gamma t_{\max}^2 = \alpha - \frac{\beta^2}{4\gamma}$. The value of t when no cases will be achieved is the larger root of $m(t) = 0$, i.e., $t_0 = (-\beta + \sqrt{\beta^2 - 4\alpha\gamma})/2\gamma$. With OLS estimators $\widehat{\theta} = (\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma})^{\intercal}$, we estimate

$$\widehat{t}_{\max} = \frac{\widehat{\beta}}{-2\widehat{\gamma}}, \quad \widehat{m}(\widehat{t}_{\max}) = \widehat{\alpha} - \frac{\widehat{\beta}^2}{4\widehat{\gamma}}, \quad t_0 = \frac{-\widehat{\beta} + \sqrt{\widehat{\beta}^2 - 4\widehat{\alpha}\widehat{\gamma}}}{2\widehat{\gamma}} \tag{2}$$

The standard errors are available from the delta method. We have:

$$\mathrm{avar}(\widehat{t}_{\max}) = \begin{pmatrix} 0 - \frac{1}{2\gamma} \, \frac{\beta}{2\gamma^2} \end{pmatrix} \mathrm{avar}(\widehat{\theta}) \begin{pmatrix} 0 \\ -\frac{1}{2\gamma} \\ \frac{\beta}{2\gamma^2} \end{pmatrix},$$

$$\mathrm{avar}(\widehat{m}(\widehat{t}_{\max})) = \begin{pmatrix} 1 - \frac{\beta}{2\gamma} \, \frac{\beta^2}{4\gamma^2} \end{pmatrix} \mathrm{avar}(\widehat{\theta}) \begin{pmatrix} 1 \\ -\frac{\beta}{2\gamma} \\ \frac{\beta^2}{4\gamma^2} \end{pmatrix},$$

where $\mathrm{avar}(\widehat{\theta})$ is the covariance matrix of the least squares parameter estimates. Results are available from the author upon request.