F500 Problem Set 1

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1. First obtain daily price data on a stock index and two individual stocks from a market of your choice. The method for assignment is given below. The calculations can be performed in Excel and/or Eviews, but also in other software packages, as you prefer.

   (a) Compute the sample statistics of the stock return (computed from the daily closing price) series, i.e., the mean, standard deviation, skewness and kurtosis. You may ignore dividends and just focus on capital gain.

   (b) Compute the first 20 autocorrelation coefficients and test whether the series is linearly predictable or not.

   (c) Does it make a difference whether you compute returns using log price differences or as actual return?

2. The (equal weighted) moving average filter of a series X_t is defined as

$$SMA_t^k = \frac{X_t + X_{t-1} + \ldots + X_{t-k}}{k},$$

   where k is the number of lags to include, sometimes called a bandwidth parameter. For daily stock prices, common values include 5, 10, 20, 50, 100, and 200. Compute the SMA for the series. The exponential weighted average is defined as

$$EWMA_t = \alpha X_t + (1 - \alpha) EWMA_{t-1},$$

   where $EWMA_1 = X_1$ and $\alpha \in (0, 1)$. Can relate $\alpha = 2/(k + 1)$, where k is number of time periods. These "smoothed" values are often used in trading strategies of the contrarian type, that is: buy when $X_t < SMA_t^k$ and sell when $X_t > SMA_t^k$, or moment type trading strategies, that is, buy when $X_t > SMA_t^k$ and sell when $X_t < SMA_t^k$. Comment on the efficacy of these trading strategies for your dataset. See "A Quantitative Approach to Tactical Asset Allocation" (M. Faber) http://ssrn.com/abstract=962461.

3. The so-called Bollinger bands (http://en.wikipedia.org/wiki/Bollinger_Bands) are a modification of the moving average rules that allow a margin of safety by allowing for time varying volatility. They are defined as follows:

$$\begin{aligned} BB_t^U &= SMA_t^k + 2\sigma_t \\ BB_t^L &= SMA_t^k - 2\sigma_t \\ \sigma_t &= std(X_t, X_{t-1}, \ldots, X_{t-k}) \end{aligned}$$

Compute the Bollinger bands for your data series and compare the trading strategies: buy when $X_t < BB_t^L$ and sell when $X_t > BB_t^L$, or moment type, that is, buy when $X_t > BB_t^U$ and sell when $X_t < BB_t^L$.

4. The principle behind the moving average filter can also be called "rolling window". Apply the rolling window principle to the calculations of question 1. That is, for $k = 252$ (corresponding to an annual "window") compute the statistics of 1a,b using the sample $\{r_t, ..., r_{t-k}\}$ or $\{R_t, ..., R_{t-k}\}$. Comment on how the results vary with t.

5. Suppose that true returns $r_1, \ldots, r_T$ are recorded as

$$0, \ldots, 0, r_1 + \cdots + r_k, 0, \ldots, r_{k+1} + \cdots + r_{2k}, \ldots, 0, \ldots, 0, r_{T+1-k} + \cdots + r_T,$$

where $T = j \times k$. Let $\widetilde{r}_t$ denote the typical member of this sequence. Compare

$$\overline{r} = \frac{1}{T}\sum_{t=1}^{T} r_t, \quad s_r^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \overline{r})^2, \gamma_r(s) = \frac{1}{T-s}\sum_{t=s+1}^{T}(r_t - \overline{r})(r_{t-s} - \overline{r})$$

with

$$\frac{1}{T}\sum_{t=1}^{T}\widetilde{r}_t, \frac{1}{T-1}\sum_{t=1}^{T}(\widetilde{r}_t - \overline{\widetilde{r}})^2, \frac{1}{T-s}\sum_{t=s+1}^{T}(\widetilde{r}_t - \overline{\widetilde{r}})(\widetilde{r}_{t-s} - \overline{\widetilde{r}}).$$

What can you say in general? What happens when $k = T$ and $j = 1$? How does this relate to the non trading model considered in CLM?

6. Show that in the Roll model

$$\text{cov}((\Delta P_t)^2, (\Delta P_{t-1})^2) = 0.$$

How would you go about testing this implication of the Roll model? For the data you obtained in the first exercise, check whether this implication seems reasonable when prices or log prices are used.

7. Extend the Roll model to allow the spread s to vary over time so that

$$P_t = P_t^* + \frac{1}{2} I_t s_t,$$

where I_t is as before. Suppose that $s_1, \ldots, s_T$ are i.i.d. independent of $I_1, \ldots, I_T$ with mean μ_s and variance σ_s^2. Calculate

$$\text{cov}((\Delta P_t, \Delta P_{t-1}) \quad ; \quad \text{cov}((\Delta P_t)^2, (\Delta P_{t-1})^2).$$

Now suppose that $s_1, \ldots, s_T$ are deterministic and dont vary. Show that

$$\text{cov}((\Delta P_t, \Delta P_{t-1}) = -\frac{1}{4} s_{t-1}^2$$

Typically, we expect spreads to widen at the open and the close of a market, what should this say about the predictability of returns during the day?

8. Suppose you want to apply event study methodology to detect insider trading. Explain some of the issues that may be involved. Specifically, what type of data would you need? What event window would you choose? What econometric methods would you use? You may focus on the country that you chose in the first exercise sheet. For comparison, read the article Wong (2002).