F500 Revision Session - 2019

Dr Tom Auld

2014

A4. Suppose that (excess) returns obey a linear K-factor model

$$Z_{it} = \mu_i + \sum_{j=1}^{K} b_{ij} f_{jt} + \varepsilon_{it}$$

where ε_{it} is an (iid over t) idiosyncratic error term with

$$\mathrm{cov}(\varepsilon_{it}, f_{js}) = 0 \quad ; \quad E(\varepsilon_{it}\varepsilon_{js}^{\intercal}) = \begin{cases} \sigma_{ij} & \text{if } t = s \\ 0 & \text{else} \end{cases} .$$

You observe the panel $\{Z_{it},\, i = 1, \ldots, N,\, t = 1, \ldots, T\}$.

(a) The factors are themselves not directly observed. Describe how you might estimate the factors $\{f_{jt},\, j = 1, \ldots, K\}$ in two cases:

i. The time series T is large compared with the number of assets N;

ii. The time series T is short compared with the number of assets N.

(b) Suppose now that factor returns are assumed to be observed. How would you test the Arbitrage Pricing Theory?

B1. Explain Campbell's Approximate Model of stock prices

$$\begin{aligned} p_t &= \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j \left[(1-\rho)\, d_{t+1+j} - r_{t+1+j}\right] \\ &= \frac{k}{1-\rho} + (1-\rho) \sum_{j=0}^{\infty} \rho^j E_t d_{t+1+j} - \sum_{j=0}^{\infty} \rho^j E_t r_{t+1+j}, \end{aligned}$$

where p_t is log stock prices, d_t is log dividends, and r_t is stock returns, while E_t denotes expectation conditional on all information at time t. Specifically, explain how it is derived and what is the constant ρ? Explain how this equation might be used to explain the observed variability of stock prices. Explain how one might use Vector Autoregressions to obtain estimates of $E_t r_{t+1+j}$.

2015

2. Suppose that daily stock returns satisfy

$$r_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \omega + \gamma r_{t-5}^2,$$

where ε_t is iid standard normal.

(a) Is this model consistent with the Semi-Strong Efficient Markets Hypothesis?

(b) Is this model consistent with the Weak Form Efficient Markets Hypothesis?

(c) Is this model consistent with the "stylized empirical fact" that

$$\text{cov}(r_t^2, r_{t-k}^2) > 0$$

for all $k = 1, 2, \ldots$

(d) Is this model consistent with the "stylized empirical fact" that

$$\text{cov}(r_t^2, r_{t-k}) < 0$$

for all $k = 1, 2, \ldots$

2016

A3. Suppose that the dividend/price ratio x_t and stock returns r_t obey the following predictive regression

$$\begin{aligned} r_{t+1} &= \beta x_t + \varepsilon_{t+1} \\ x_{t+1} &= \rho x_t + \eta_{t+1}, \end{aligned}$$

where the error terms are iid with mean zero and coviarance matrix given below

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \overbrace{\sim}^{iid} 0, \begin{pmatrix} \sigma_{\varepsilon\varepsilon} & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_{\eta\eta} \end{pmatrix}.$$

Empirically we find that β is quite large and statistically significant but returns are almost uncorrelated over time. How can that be?