An Introduction to Financial Markets and their Empirical Analysis

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Why Do We Have Financial Markets?

- The purpose of financial markets is to channel funds from savers to borrowers.
  - Individuals have a preference for **consumption smoothing**, both
    - ★ intertemporal (saving and dissaving over time so that consumption is less affected by time-varying income), and
    - ★ intratemporal, i.e., across states of nature (to avoid or mitigate the effects of disaster or large price swings).
- Diversification of **idiosyncratic** (individual specific) risks enables the mobilization of society's savings for long-term projects (which may often be the most productive).

- There is economic efficiency and growth if entrepreneurs with productive ideas/projects borrow from savers with inferior investment opportunities of their own (households are typically net savers; firms are net borrowers; government can be either, it depends on revenues versus expenditures).
- To provide the **insurance** or **hedging** demanded by some requires that someone, perhaps called **speculators**, to take the other side thereby enabling the **risk transfer**.
- Ultimately these transactions can increase welfare, so financial markets are not necessarily in total a **zero sum game**.

- Financial markets are physical or virtual venues where demand meets supply.
    - Ideally, they guide scarce resources towards their most productive use.
    - Investors analyze companies and bid prices up or down, which influences the cost of capital (both debt and equity). Ultimately, this determines which companies will live and which will die.
    - Ideally, this enables efficient **risk management** - the slicing and dicing of risks and their transfer to parties most willing to bear them - which improves household welfare and reduces firms' cost of capital.

- A key feature of security markets is that they allow the separation of ownership and management.
  - A small business can be owner-managed.
  - A large corporation has capital requirements that exceed the possibilities of single individuals, and has possibly hundreds of thousands of shareholders. They elect the board of directors, which hires managers who ideally run the business efficiently and maximize its value for the shareholders.
- In principle, this separation ultimately leads to improvement of economic efficiency and welfare. However, conflicts of interests arise often in financial markets, and this can lead to negative outcomes.

Do Financial Markets Function Well?

- The efficiency of the capital allocation process matters. A country can save and invest a large fraction of its output but if the financial system allocates it inefficiently, a shortfall in growth and welfare ensues.
- What can go wrong? Some examples of inefficiencies might include:
  - preferential flow of credit to Party members' companies,
  - poor funding options for small and medium sized enterprises (SMEs),
  - poor legal systems such as those that allow **insider trading** with impunity. There is some evidence that enforcement of insider trading laws reduces the cost of equity/capital, Bhattacharya and Daouk (2002).
  - Market manipulation
  - Bubbles, frenzies, crashes

- The **Principal-agent problem** (owners v managers):
  - Shareholders want the most productive projects, i.e. those with the highest positive ‘net present value’, to be undertaken.
  - Instead, managers may pursue their own interests: empire building, take excessive risks to generate short-term profits and ignore long-term consequences, keep inefficient suppliers in exchange for kickbacks, engage in corporate book cooking, etc.
- Some well known recent examples include: WorldCom, Enron, and Parmalat. Arthur Andersen, the demised accounting company, received more income from consulting for Enron during the 1990s than from auditing it. Enron used special purpose entities/vehicles to get debt off its books and the auditor might have been lenient to protect its consulting profits.
- In principle, boards of directors may force underperforming managers out. They may design compensation contracts that align incentives, or they may not. Security analysts, large shareholders and creditors monitor the management, and they can sometimes affect the direction it takes.

- Some other examples of conflicts of interest include:
  - optimistic security research in exchange for investment banking business (hence information barriers between corporate finance operations and retail or trading business);
  - Trading by brokers against or ahead of clients (**front running**);
  - Insider trading where insiders or their proxies trade ahead of product news, earnings announcements, and merger and acquisition deals.
- The financial crisis of the late 2000s demonstrated a range of weaknesses in the financial markets architecture including **Moral hazard**:
  - Executives faced incentives towards taking excessive risks: big profits yield big bonuses, big losses do not coincide with negative bonuses.
  - Government bailouts were needed to stabilize the system with large future fiscal consequences.
  - Additional regulation and supervision were introduced. Many fines were issued to investment banks for misconduct. Turner (2009) provides some analysis of what happened.

We can classify financial markets into the following:

- **Money Markets** - debt instruments with maturity $<$ 1 year;
- **Bond Markets** - debt instruments with maturity $\geq$ 1 years;
- **Stock/Equity Markets** - shares of listed companies;
- **Foreign Exchange Markets** - currency pairs;
- **Derivatives Markets** - futures contracts, options;
- **Commodity Markets** - pork bellies, copper etc.

According to the World Federation of Stock Exchanges (FESE), the 10 biggest stock markets in the world by market capitalization in (US$ millions) at the end of 1999 and 2010 are shown below

Rank	Exchange 1999	Dollar	Exchange 2010	Value
1	NYSE	11,437,597.3	NYSE Euronext (US)	13,394,081.8
2	Nasdaq	5,204,620.4	NASDAQ OMX	3,889,369.9
3	Tokyo	4,463,297.8	Tokyo SE Group	3,827,774.2
4	London	2,855,351.2	London SE Group	3,613,064.0
5	Paris	1,496,938.0	NYSE Euronext (Europe)	2,930,072.4
6	Deutsche Börse	1,432,167.0	Shanghai SE	2,716,470.2
7	Toronto	789,179.5	Hong Kong Exchanges	2,711,316.2
8	Italy	728,240.4	TSX Group	2,170,432.7
9	Amsterdam	695,196.0	Bombay SE	1,631,829.5
10	Switzerland	693,133.0	National Stock Exchange India	1,596,625.3
10=			BM&FBOVESPA	1,545,565.7

Table: Stock exchanges ranked by value traded

Financial Returns

- The concept of return is central to finance
  - This is the benefit of holding the asset that accrues to its owner.
- The return depends on the price of the asset and the holding period considered.
- In classical economics one is taught about the centrality of the price system and the law of one price. In practice, prices are formed in markets of different types (auction markets, dealer markets, etc.). They could be transaction prices or merely quoted prices. They may be the weighted averages of prices of different transactions.

We suppose that prices P are observed at some times t. Some assets also pay the holders a dividend, which should be included in the definition of the total return.

Definition

The simple gross return between times s and t is calculated as

$$\mathcal{R}_{s:t} = \frac{P_t + D_t}{P_s},$$

where D_t are any dividends paid out during the holding period $[s, t]$. The simple net return is defined

$$R_{s:t} = \mathcal{R}_{s:t} - 1.$$

- The gross return is non-negative because prices are non-negative, but the net return can be negative, but not less than -1.
- Total return can be divided into the **dividend return** D_t / P_s and the **capital gain** P_t / P_s.

Definition

The continuously compounded return or log return over the interval $[t-1, t]$ is

$$r_t \equiv \log \mathcal{R}_t = p_t - p_{t-1},$$

where $p_t \equiv \log P_t$. The multiperiod log return is

$$\begin{aligned} r_{t:t+k} &= \log \mathcal{R}_{t:t+k} = \log(\mathcal{R}_{t-k+1} \times \mathcal{R}_{t-k+2} \times \ldots \times \mathcal{R}_t) \\ &= r_{t-k+1} + r_{t-k+2} + \ldots + r_t. \end{aligned}$$

Using logarithmic returns it is easy to annualize daily returns. Common practice is to multiply average daily returns by 252 to get an average annual return, since there are roughly 252 trading days in the NYSE trading year (This is based on the calculation $(365.25 \times (5/7)) - 9$, where there are currently 9 public holidays per year. The current primary opening hours of NYSE are 9.30am-4pm, although there are three days close to holiday periods with shorter hours 9.30am-1pm.).

Example

We consider the daily S&P500 index from 19500103 to 20170321, a total of $n = 16913$ daily prices. The dominant feature of the price sequence shown below is the substantial increase over time. We have $P(1) = 16.66$ and $P(n) = 2344.02$ and so the full return over the period is

$$R_{1:n} = \frac{P(n) - P(1)}{P(1)} = 139.0.$$

On the other hand using the logarithmic definition of multiperiod return

$$r_{1:n} = \log(P(n)) - \log(P(1)) = 4.947.$$

This shows the substantial difference between the actual return and the logarithmic return (in this case just the capital gain) over the long horizon. The annualized returns are $r_{year} = 255 \times 4.947/16913 = 0.0746,$ whereas the annualized $R_{year} = \log(139)/66 = 0.075,$ which are almost identical.

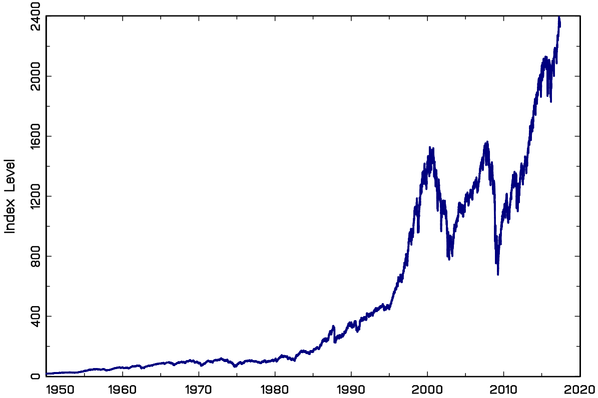


Figure: Level of the S&P500 index

- By comparison, annual US Gross Domestic Product rose from \$0.300 trillions in 1950 to \$18.569 trillions in 2016, a growth of around 61 fold.
- The consumer price index (CPI) rose from 24.1 in 1950 to 240.01 in 2016 (annual averages), around a 10 fold increase.
- So the growth of the level of the stock market (and this indeed only includes capital gain) is substantial indeed over the long horizon.
- For historical reference, Malthus (1798) argued that population growth was exponential (R), whereas food production technology grew at a linear rate (r).

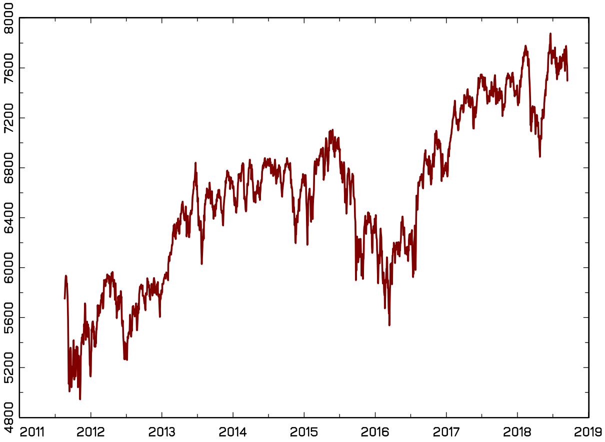


Figure: FTSE100 Index: Total Capital Gain (20110718-20180815) 0.30334045

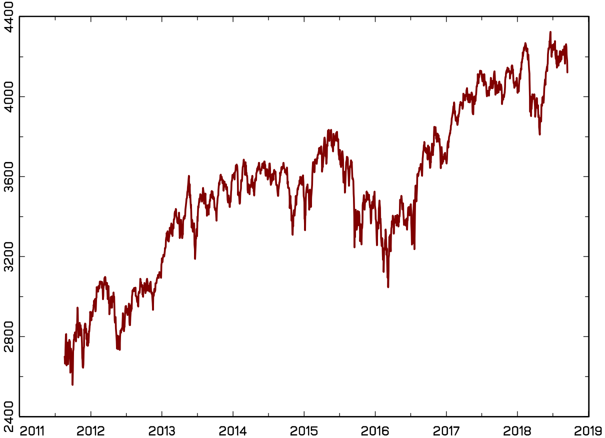


Figure: FTSE Allshare: Total Capital Gain 0.52728728

Figure: Shanghai Composite Index: Total Capital Gain -0.033170139

Figure: DAX Index: Total Capital Gain 0.71119118

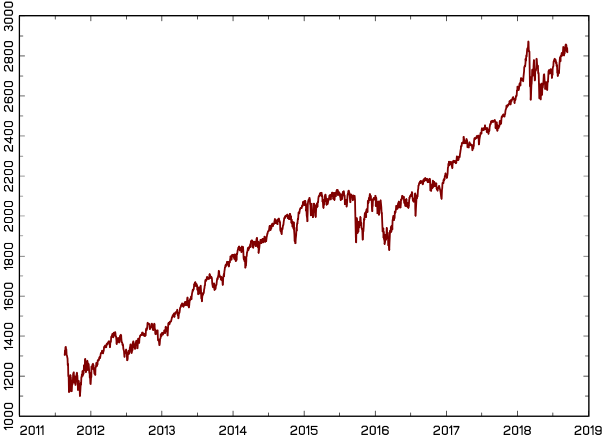


Figure: S&P500 Index: Total Capital Gain 1.1589426

We show the time series plot of daily returns on the S&P500 over the same period

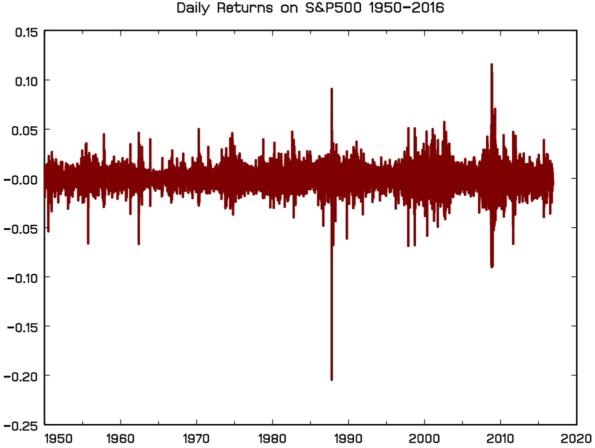


Figure: Daily return on the S&P500 daily index

- The main features are the extreme negative value in October of 1987, which was around 21 standard deviation units.
- Historically, many authors assumed that returns are normally distributed. But if returns were normally distributed, the likelihood of such an event would be **machine zero**, that is so small as to be indistinguishable from zero by most computer calculations.
- This is why a lot of research has gone into developing models for returns in which such events are not so unlikely and can be given some rationale.

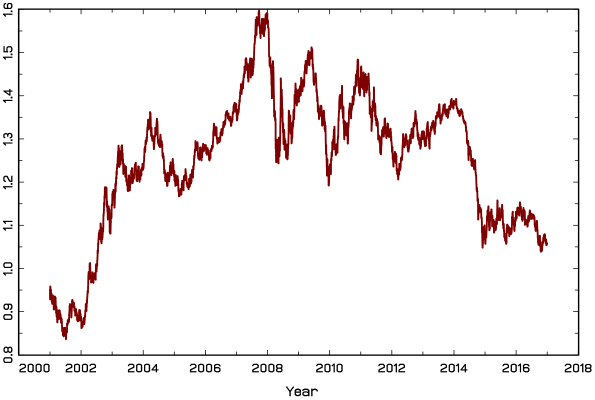


Figure: Euro dollar daily exchange rate

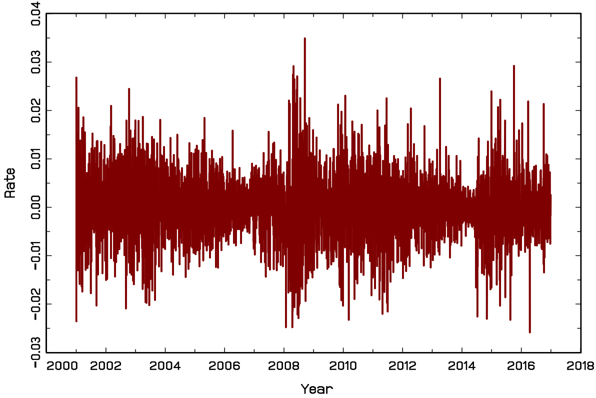


Figure: Return on the euro/usd daily exchange rate

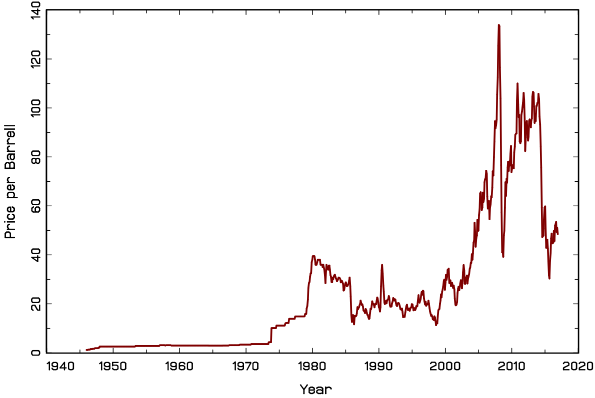
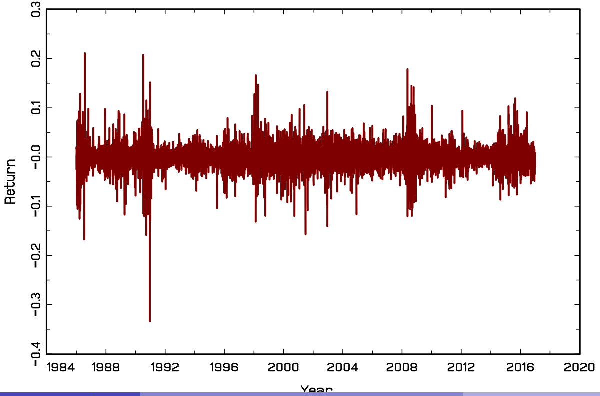


Figure: Price of West Texas Oil, Monthly frequency

The price of oil has been an important bellwether of industrialized economies. The price of oil was pretty low and varied very little from year to year until the OPEC price hike of 1974 as shown in Figure 10. Thereafter the price has been determined partly by financial market activity and partly by the production decisions of OPEC and other nations. The daily return series (from 1986) shown in Figure 11 looks very similar to the stock return series



Risk Aversion and Expected Utility Theory

- Economists wish to study rational choice among alternative risky prospects.
- We represent the random outcome of a gamble by the random variable X.

Definition

A Fair gamble is a gamble that is (actuarially) fair, i.e., is zero in expectation, $E(X) = 0$.

- Most gambles are unfair, lotteries, roulette, etc

Example

Roulette. Consider a casino roulette wheel with 37 slots, 18 red, 18 black, and 1 void, and suppose that you bet on black. We can describe the nett gain per play of the Casino through the random variable X, where

$$X = \begin{cases} 1 & 19/37 \\ -1 & 18/37. \end{cases}$$

The expected rate of return for the Casino

$$E(X) = \frac{19}{37} - \frac{18}{37} = \frac{1}{37}.$$

This gamble is not fair. The expected rate of return for the Casino is $1/37$ per $ play.

Example

St Petersburg Paradox. Consider the following game of chance: you pay a fixed fee to enter and then a fair coin is tossed repeatedly until a tail appears, ending the game. The pot starts at 1 dollar and is doubled every time a head appears. You win whatever is in the pot after the game ends. Thus you win 1 dollar if a tail appears on the first toss, 2 dollars if a head appears on the first toss and a tail on the second, 4 dollars if a head appears on the first two tosses and a tail on the third etc. What would be a fair price P to pay for entering the game? The expected value is

$$P = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 4 + \ldots = \infty.$$

This seems counterintuitive. Certainly, no-one is going to pay this. Bernoulli solved this problem by computing the logarithm of earnings (a utility function) instead and taking its expectation

$$\frac{1}{2} \times \log 1 + \frac{1}{4} \times \log 2 + \frac{1}{8} \times \log 4 + \ldots = \sum_{k=1}^{\infty} \frac{k-1}{2^{k-1}} = \log 2 < \infty.$$

- We now consider **Expected Utility Theory**. Individuals maximize expeted utility *EU* when they choose between risky outcomes.

Definition

An individual is risk-averse if she dislikes a fair gamble (she will not accept it for free). Similarly, she is risk-seeking/loving if she likes it, and risk-neutral if she is indifferent about it.

Theorem

A decision maker with utility function U is locally (globally) risk-averse if and only if U is strictly concave at a given wealth level (all wealth levels), i.e., $U' > 0$ and $U'' < 0$

- A risk averse individual requires some compensation to take gambles. Suppose their initial wealth is W and X is a gamble with random outcome.

Definition

The Certainty equivalent (CE) of gamble X is defined by

$$E\left(U(W+X)\right)=U(W+CE).$$

It is the amount of money that gives the individual the same welfare as the gamble.

- If the individual is risk-averse, then

$$CE<E(X).$$

- The risk premium is then

$$\Pi=E(X)-CE.$$

Definition

The Arrow-Pratt measure of absolute and relative risk aversion are

$$A(W) = -\frac{U''(W)}{U'(W)}, \mathcal{R}(W) = -W \times \frac{U''(W)}{U'(W)}.$$

Risk aversion is driven by the fact that marginal utility decreases with wealth.

Example

CRRA (constant relative risk aversion) utility:

$$U(W) = \begin{cases} \frac{W^{1-\gamma}-1}{1-\gamma} & \text{for } \gamma \neq 1 \\ \log W & \text{for } \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion, $\mathcal{R}(W) = \gamma$

Some Key Questions

1. Do financial markets work well?
    1. Are they fair?
    2. Are they orderly?
    3. Are they efficient?
    4. Are they transparent?
2. How do financial markets work in detail and how do economists model them?
3. How to measure risk and return? How to trade off risk versus return?
4. Idiosyncratic risk versus systematic risk. Diversification

- An **orderly market** is a market in which supply and demand for a product or security are roughly equal. Because of this, orderly markets tend not to be volatile and prices tend to reflect the true value of the product or security. On a (stock) exchange, this supposes that bid prices and ask prices are provided consistently, and there are few price fluctuations and no large ones. The presence of an orderly market on an exchange is often due to both diligence on the part of market makers and the existence of an orderly market in the sense of demand/supply balance.
- The **fair market value** is the price at which an asset would change hands between a willing buyer and a willing seller, neither being under any compulsion to buy or to sell and both having reasonable knowledge of relevant facts.

Efficient Markets Hypothesis

Definition

Fama (1970, JoF): A market in which prices always fully reflect available information is called efficient (EMH)

- If prices are predictable $\Rightarrow$ opportunities for superior returns (free lunch) $\Rightarrow$ will be competed away immediately by a lot of hungry traders $\Rightarrow$ unpredictable random walk
  - If a security is believed to be underpriced, buying pressure $\Rightarrow$ jump up to a level where no longer thought a bargain
  - If a security is believed to be overpriced, (short-)selling pressure $\Rightarrow$ jump down to a level where no longer thought too expensive
- As a result, market forces respond to news quckly and make prices the best available estimates of fundamental values, i.e. values justified by likely future cash flows and preferences of investors/consumers

We distinguish among three forms of market efficiency depending on the information set with respect to which efficiency is defined

1. **Weak form.** (1) Information from historical prices are fully reflected in the current price; (2) One can't earn **abnormal profits** from trading strategies based on past prices alone.
2. **Semi strong form.** (1) All public information (past prices, annual reports, quality of management, earnings forecasts, macroeconomic news, etc.) is fully reflected in current prices; (2) One can't earn abnormal profits from trading strategies based on public information.
3. **Strong form.** (1) All private and public information is fully reflected in current prices; (2) One can't earn abnormal profits from trading strategies based on all information including public and private.

$$Strong \Longrightarrow Semi\ strong \Longrightarrow Weak$$

Technical analysts

- Chartists try to identify regularity of some patterns in stock prices, hoping to exploit them and profit. They believe patterns are repeated in prices. e.g. Head and Shoulders

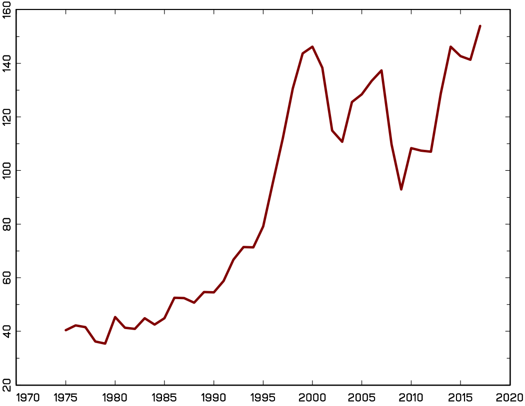
  This one predicts that future prices decline.

- Lo and Hasanhodzic (2010) connected the analysis of chartists to nonlinear time series analysis. They show how to convert observed price history into a numerical score that identifies say "head and shoulderness". They show that there is some basis to their work, but provide the tools to replace them by automated systems.

Fundamental analysts

- They estimate future cash flows from securities and their riskiness, based on analysis of company-relevant data such as balance sheets as well as the economic environment in which it operates, to determine the proper price of securities. Graham and Dodd class book on investing espoused by Warren Buffett.
- For example, buy stocks with low Price/Earnings (P/E) and sell high P/E stocks
- Ratio of Stock market value to GDP (Warren Buffet)

- Warren Buffet: Total market cap to GDP (%). Shows the US might be in a bubble



Two academic critiques of EMH

- Grossman and Stiglitz (1980, AER) point out that if information collection and analysis are costly, there must be compensation for such activity in terms of extra risk-adjusted returns, otherwise rational investors would not incur such expenses. Therefore, **Markets cannot be fully informationally efficient**, rather there is an ‘equilibrium degree of disequilibrium’. Weak form may approximately hold but semistrong harder to justify.
- Shleifer and Vishny (1997, JF). Textbook arbitrage is a costless, riskless and profitable trading opportunity; in practice it is usually costly and risky. Also is conducted by a small number of highly specialized professionals using other people’s capital (principal/agent relationship). If the misspricing temporarily worsens, investors/clients may judge the manager as incompetent and refuse to provide additional capital (margin call) and make withdrawals, thus forcing him to liquidate positions at the worst time. He loses performance fees, and perhaps a career ender). Therefore, a rational specialized arbitrageur may not take the trades.

Empirical Testing of EMH:

- Joint Hypothesis problem. Any test of weak form EMH must assume an equilibrium asset pricing model that defines 'normal' security returns against which investor returns are measured. If we reject the hypothesis that investors can't achieve superior risk-adjusted returns, we don't know if markets are inefficient or if the underlying model is misspecified. Therefore, we can never reject EMH.
- We will assume that the expected return $\mu_t = \mu$ is constant or its variation is small. This can be justified if the frequency is high and or risk aversion is small.

The Random Walk Hypothesis

Definition

The random walk

$$X_t = \mu + X_{t-1} + \varepsilon_t,$$

where $X_t = p_t$ or $X_t = P_t$. Three general assumptions:
(1) RW1: $\varepsilon_t \sim IID$; $E\varepsilon_t = 0$;
(2) RW2: ε_t *independent over time*; $E\varepsilon_t = 0$;
(3) RW3: For all k : $\operatorname{cov}(\varepsilon_t, \varepsilon_{t-k}) = 0$

The traditional model for stock prices, it says that prices evolve randomly and that μ is constant expected return. Historically, μ was often assumed to be zero and ε_t normally distributed, even stronger than (1). We also consider the more natural assumption of MDS

Definition

A martingale is a time-series process X_t obeying

$$E\left[X_{t+1} \mid X_t,\ X_{t-1},\ ...\right] = X_t$$

or equivalently, call $\varepsilon_{t+1} = X_{t+1} - X_t$ a martingale difference sequence (MDS) if

$$E\left[\varepsilon_t \mid X_{t-1},\ X_{t-2},\ ...\right] = 0.$$

This corresponds with the notion of a fair game: If you toss a coin against opponent and bet successively at fair odds with initial capital X_0, current capital X_t is a martingale.

This is the case that $\mu = 0$; More generally, we might assume that

$$\varepsilon_{t+1} = X_{t+1} - X_t - \mu$$

is a martingale difference (MDS). Increments are essentially unpredictable given past information.

Martingale property implies that

$$\mathrm{cov}(\varepsilon_t, g(X_{t-1},\ X_{t-2},\ ...)) = 0$$

for any (measurable) function g.
In particular returns are uncorrelated (RW3) but also

$$\mathrm{cov}(r_t, g(r_{t-1}, \ldots, r_{t-p})) = 0$$

So stronger condition than RW3. Call it RW2.5.

Theorem

Provided $E\varepsilon_t^2 \leq C < \infty$,

$$RW1 \Longrightarrow RW2 \Longrightarrow RW2.5 \Longrightarrow RW3 : \mathrm{cov}\left(\varepsilon_t, \varepsilon_{t-k}\right) = 0$$

- We test the implication of the weak form EMH that demeaned returns are uncorrelated.
- The population autocovariance and autocorrelation functions of a stationary series Y_t

$$\gamma_s = \mathrm{cov}(Y_t, Y_{t-s}) = E\left[\left(Y_t - EY_t\right)\left(Y_{t-s} - EY_{t-s}\right)\right]$$

$$\rho_s = \frac{\gamma_s}{\gamma_0}$$

for $s = 0, \pm 1, \pm 2, \ldots$ Take $Y_t = r_t$ or R_t. The efficient markets hypothesis says that $\gamma_s, \rho_s = 0 \quad$ for all $s \neq 0$.

Can estimate these quantities by the sample equivalents

$$\widehat{\gamma}_s = \frac{1}{T} \sum_{t=s+1}^{T} (Y_t - \overline{Y})(Y_{t-s} - \overline{Y})$$

$$\widehat{\rho}_s = \frac{\widehat{\gamma}_s}{\widehat{\gamma}_0}.$$

- Assume further that Y_t is i.i.d. It can be shown that for any k, as $T \to \infty$

$$\sqrt{T}\widehat{\rho}_k \Longrightarrow N(0,1)$$

under the null hypothesis of no correlation.

- Therefore, you can test the null hypothesis by comparing $\widehat{\rho}_k$ with the so-called 'Bartlett intervals'

$$[-z_{\alpha/2}/\sqrt{T}, z_{\alpha/2}/\sqrt{T}],$$

where z_α are normal critical values. Values of $\widehat{\rho}_k$ lying outside this interval are inconsistent with the null hypothesis. Literally, this is testing the hypothesis that $\rho_k = 0$ versus $\rho_k \neq 0$ for a given k.

- Under the alternative hypothesis

$$\sqrt{T}\widehat{\rho}_k \xrightarrow{P} \infty$$

for at least one k.

- In fact, under this assumption we have

$$\sqrt{T}(\widehat{\rho}_1, \ldots, \widehat{\rho}_P)^{\intercal} \Longrightarrow N(0, I_P).$$

The Box–Pierce Q statistic

$$Q = T \sum_{j=1}^{P} \widehat{\rho}_j^2$$

can be used to test the joint hypothesis that $\rho_1 = 0, \ldots, \rho_P = 0$ versus the general alternative. We have

$$Q \Longrightarrow \chi_P^2$$

under the null hypothesis, so reject when $Q > \chi_P^2(\alpha)$ for an α-level test.

- CLM results. 19620703-19941230, Daily, weekly, monthly. CRSP value weighted and equal weighted indexes. A sample of 411 individual securities from the CRSP database.
- Table 2.4. Index results
  - Positive (first lag) autocorrelation for daily indexes (0.1-0.43) which are significant using the iid standard errors $1/\sqrt{T}$. Statistically significant Q_5 and Q_{10}.
  - Weaker at weekly and monthly horizon.
  - Weaker for value weighted versus equal weighted.
- Individual Stocks
  - Small negative autocorrelation for individual stocks at daily horizon.
  - How to explain the different results between individual stocks and index? Lead lag relations between large and small stocks (Explained by cross-correlation)
- Results are not stable across subperiods

Trading Strategy Based Evidence

- We have so far emphasized statistical evidence and statistical criteria to judge the presence or absence of predictability. We now consider whether such predictability can yield a profit, and how large a profit it might yield, Lo and MacKinlay (1990).
- Consider a set of assets with one period returns $\{R_{it},\ i = 1, \ldots, n\}$. Define the equally weighted portfolio with return in period s given by $\overline{R}_s = \sum_{i=1}^{n} R_{is} / n$. Consider the following portfolio weights at time s,

$$w_{is} = \frac{1}{n}\left(R_{i,s} - \overline{R}_s\right).$$

  This can be considered a **momentum strategy**: puts positive weight (buys) on winners and negative weight (sells) on losers. By construction the weights satisfy $\sum_{i=1}^{n} w_{is} = 0$ so this is a zero net investment at time s.
- What profit does this make in period t?

$$\pi(s:t) = \sum_{i=1}^{n} w_{is} R_{i,s:t}$$

- There are famous several papers that use essentially this methodology
  - Jegadeesh and Titman (1993) who found short-term momentum, i.e., good and bad recent performance (3-12 months) continues over time (which is consistent with positive autocorrelation and not zero as with a random walk); they considered NYSE and AMEX over the period 1965 to 1989. They considered trading strategies that selected stocks based on their returns over the past J months and then evaluated their performance over a K month holding period.
  - De Bondt and Thaler (1985, 1987) suggest on the other hand that stock prices overreact to information over the longer term, suggesting that contrarian strategies (buying past losers and selling past winners) achieve abnormal returns. They consider monthly return data for New York Stock Exchange (NYSE) common stocks for the period between January 1926 and December 1982. They show that over 3- to 5-year holding periods stocks that performed poorly over the previous 3 to 5 years achieve higher returns than stocks that performed well over the same period.

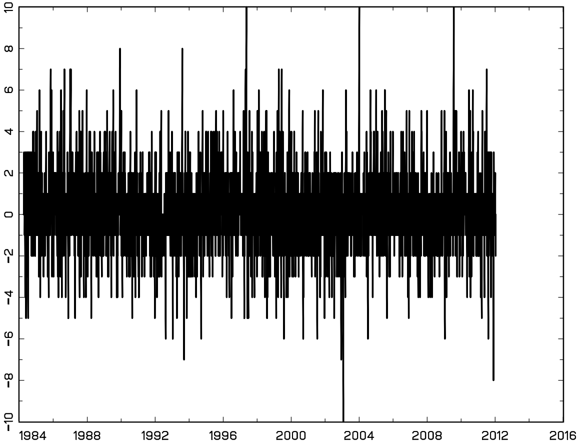
Runs Tests

- A "run" is a series of ups or downs. CLM talks about the number of runs in sample. There is a formal (nonparametric) test of RW1 using this.
- Instead we ask: How many days in a row should we see up markets or down markets?
- This is similar to the question of how many reds or blacks in a row should we see at the roulette wheel. In August 18, 1913, the colour black came up 26 times in a row at the casino in Monte Carlo. In MC roulette, there are 18 reds, 18 blacks, and a zero, so the probability of making 26 blacks in a row ex ante is

$$\left(\frac{18}{37}\right)^{26} = 7.3087 \times 10^{-9}$$

which is low, but not impossible. Since this is the maximum observed run length out of many millions presumably of rolls (since 1796) should correct for this "selection bias" in the probability calculation.

Below we show the time series of the lengths of "runs" on the daily FTSE100 return series.



Regression Based Tests (Semi Strong)

If we assume that stock returns are unforecastable (or not much) given past prices this does not preclude them being forecastable given additional information.

- Calendar effects. Suppose that

$$R_t = \mu + \beta^{\intercal} X_t + \varepsilon_t,$$

  where X_t is observed (public information) at time t deterministic seasonal dummy variables. The EMH (along with constant mean or risk premium) says that $\beta = 0$. Standard regression F test for the inclusion of X_t.

- Examples: Day of the week effect, month of the year effect, etc. Machine learning

Sample period	Mon	Tue	Wed	Thur	Fri
1950-1960	-0.00128	0.00002	0.00109	0.00093	0.00188
	(3.969)	(0.075)	(3.463)	(2.926)	(5.859)
1960-1970	-0.00158	0.00013	0.00099	0.00053	0.00089
	(5.496)	(0.465)	(3.464)	(1.862)	((3.121)
1970-1980	-0.00120	-0.00013	0.00064	0.00039	0.00068
	(3.095)	(0.356)	(1.694)	(1.011)	(1.787)
1980-1990	-0.00107	0.00113	0.00141	0.00027	0.00088
	(-2.194)	(2.384)	(2.991)	(0.556)	(1.823)
1990-2000	0.00116	0.00062	0.00088	-0.00029	0.00062
	(2.882)	(1.601)	(2.247)	(-0.748)	(1.554)
2000-2016	-0.00013	0.00056	0.00010	0.00053	-0.00028
	(0.296)	(1.331)	(0.231)	(1.228)	(0.655)

- Consider the (predictive) regression

$$R_{t+j} = \mu + \beta^{\intercal} X_t + \varepsilon_{t+j},$$

  where X_t is observed (public information) at time t or deterministic like seasonal dummy variables. The EMH (along with constant mean or risk premium) says that $\beta = 0$. Standard regression F test for the inclusion of X_t.
- Price/earnings ratio effects, dividend rate, and so on. Lots of evidence on this. Shiller website. Some econometric issues when X is very persistent process.
- Can also include nonlinear functions of observed variables to try to enhance predictability. More generally can fit nonlinear regression models.

Event Studies

- EMH says that prices react quickly to new information. Event studies try to measure how prices and returns respond to new information.
- Try to measure the impact of an event on an outcome variable Y. Classic studies: Fama, Fisher, Jensen and Roll (1969) and Brown and Warner (1980, JFE), Y stock returns.

- Financial Applications. Example of firm specific events like:
  - stock splits
  - reverse splits
  - share repurchase
  - mergers and acquisitions
  - earnings announcements
  - seasoned equity offering
  - inclusion in stock index
  - insider trading
  - Short selling restrictions
  - single stock circuit breakers
- Market wide events
  - macroeconomic announcements
  - Regulatory changes such as Dodd-Frank

Methodology

- Specifically we may compare

$$Y_{after} - Y_{before}$$

but this may be biased because there may be many things happening at the same time as the event

- Use a control group against which to measure the change in outcomes. Thus we divide into treated and control and compute the "diff in diffs"

$$\left(Y_{after}^{treated} - Y_{before}^{treated}\right) - \left(Y_{after}^{control} - Y_{before}^{control}\right)$$

- This is a general principle applied in many fields from Labor economics to medicine etc.
- The questions that need to be addressed are: how to measure Y, what is the treatment and control group, and what is before and after.

Classic Examples of an Event Study

Stock Splits

- A firm splits stock 2:1 means that it doubles the number of shares (allocating them pro rata to original owners) and halfs the price level.
- According to present value calculations this should have no effect on the valuation of the firm and on the return (which is percentage change in price) on holding the stock.
- Various other theories as to why stock splits may be beneficial

Event Definition and Alternative Hypothesis

- Types of adjustment of outcomes to an event
  - Immediate and permanent effect (including no effect) (Efficient markets)
  - Underreaction: gradual adjustment to new level
  - Overreaction: rapid adjustment that overshoots the new level and then returns to it.
- In practice, "immediate" and "permanent" must be defined with some "flexibility"
- Studies may try to allow for these possible adjustment mechanisms in the null hypothesis or may treat them as part of the alternative.
- Short term versus long term.

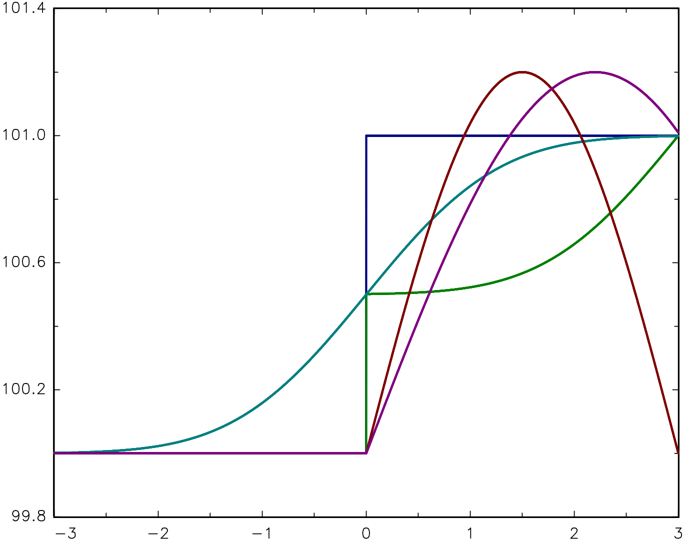


Figure: Shows examples of price trajectories around event at time 0.

Example

Dubow and Monterio (2006) develop a measure of 'market cleanliness'. The measure of market cleanliness was based on the extent to which 'informed price movements' were observed ahead of 'significant' (i.e. price-sensitive) regulatory announcements made by issuers to the market. These price movements could indicate insider trading. In that case they were looking for movements of prices before the announcement date 0.

Stock Splits

- The Announcement day (when the split is announced) is on average 52 days prior to the Ex-date (when the split is made, which is during non trading hours)
- Event window typically may start with the announcement day and end after the split day. Or may be just around the split day.
- Most splits are 2:1; Berkshire Hathaway in Jan 2010 did a 50:1 split. Effect may vary with size of split.
- Empirically, stock splits are procyclical - many stock splits at the end of a bull market

Most traders view stock splits as high potential trading opportunities. They consider splits a positive progression in value and goodwill for companies and their investors. Corporate executives use stock splits as marketing and investor relation tools. They know that stock splits make shareholders feel better and engender a sense of greater wealth.
Berkshire Hathaway (Warren Buffet's company) stock prices soars above $200,000 (Financial times, 15/08/14)

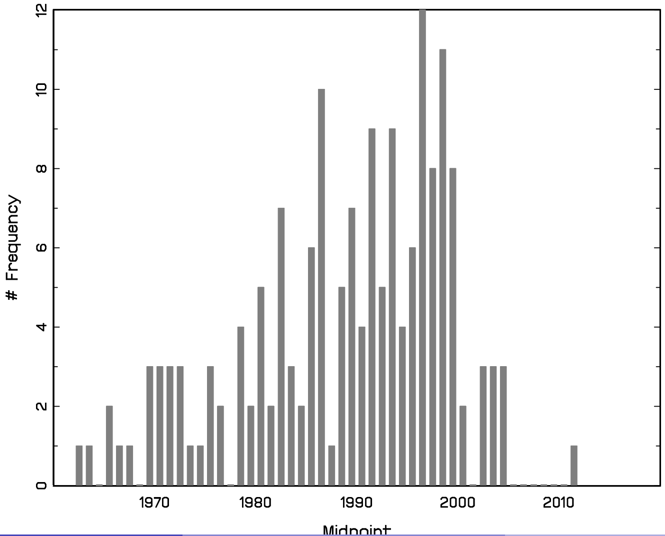
> *Shareholder eugenics might appear to be a hopeless undertaking. However, were we to split the stock or take other actions focussing on stock price rather than business value, we would attract an entering class of buyers inferior to the existing class of sellers (WB, Letter to shareholders 1983)*

A hard to trade stock encourages investors to take a long term view and locks out those more likely to trade on emotion

- Dolley (1933) studies splits between 1921-1931 and found price increases at the time of split. Short windows
- Fama et al. (1969) study. Argues that Dolley findings flawed, he did not control for price appreciation trend established prior to split. CARs, simple market model, monthly data, Key point is large window around the split date, ± 30 months. 940 splits between 1927-1959. They find:
  - CAR increased linearly up to split date and then stayed constant. That is, abnormal returns prior to the split date but not after
  - Argue that results are consistent with the semi strong form of efficiency. Most of the effect occurs a long time before the actual split date. Sample selection: firms that split tend to have had a period of high price appreciation prior to split decision. Splits tend to happen more during bull markets than bear markets.
  - They argue that split announcement signals that dividends may increase in future. Provides some evidence on this by dividing into high dividend after ex group and low dividend after ex group. Found that the high group had positive ARs after ex date, while the low group has negative ARs upto a year after the ex date.

Dow Splits

Total of 167 splits for Dow stocks (for the samples we obtained from Yahoo, which in some cases go back to 1960). When did Dow splits occur?

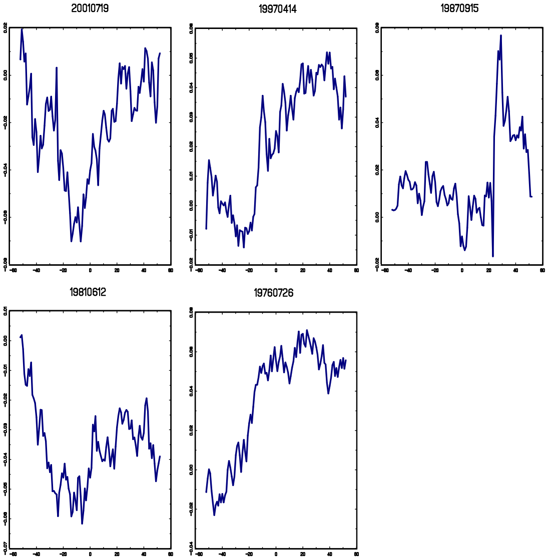


What is the typical size?

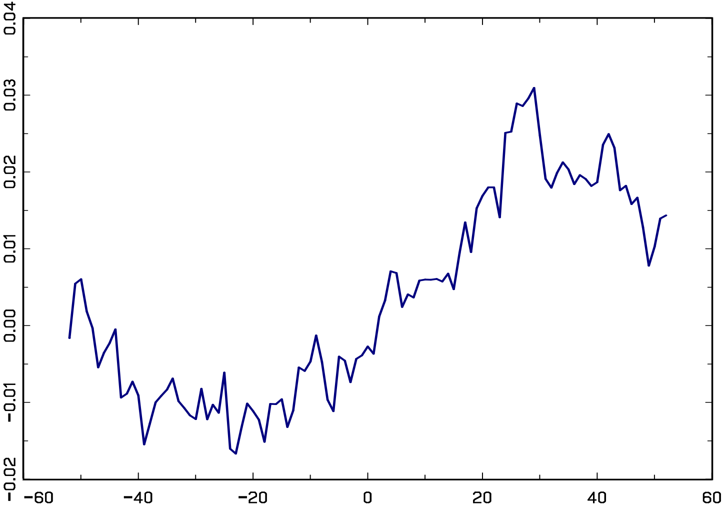
Size	Number
$<$1.5	3
1.5	30
2	119
3	12
4	3

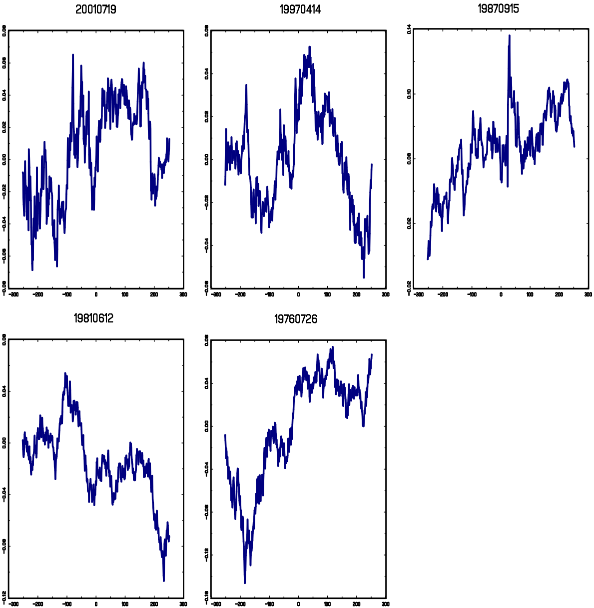
Could use as regressor in cross sectional regression of CAR

For Exxon, there were 5 splits during the sample period. Below shows the ± 52 CAR for each one

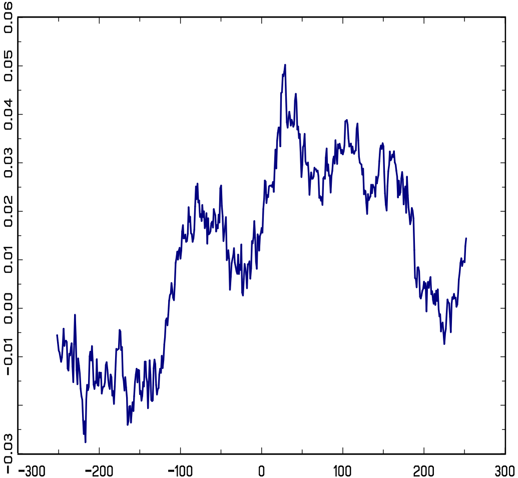


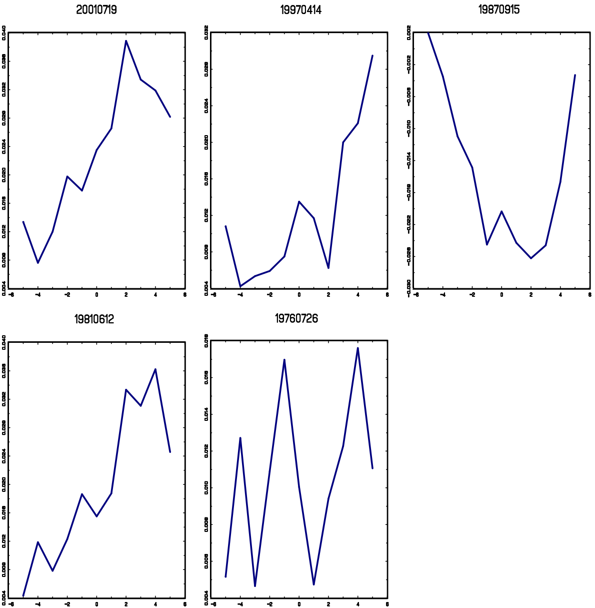
Average over splits




20010719
19970414
19870915
19810612
19760726

Average over splits




20010719
19970414
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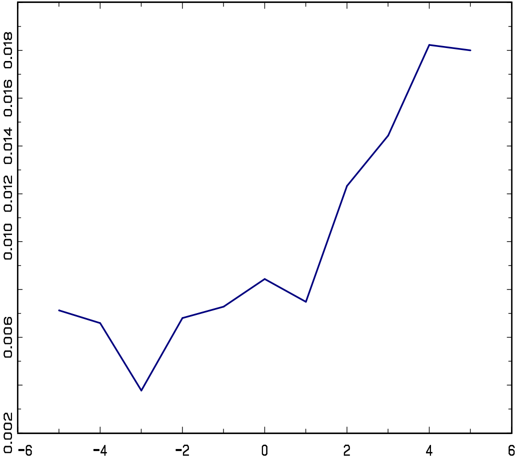
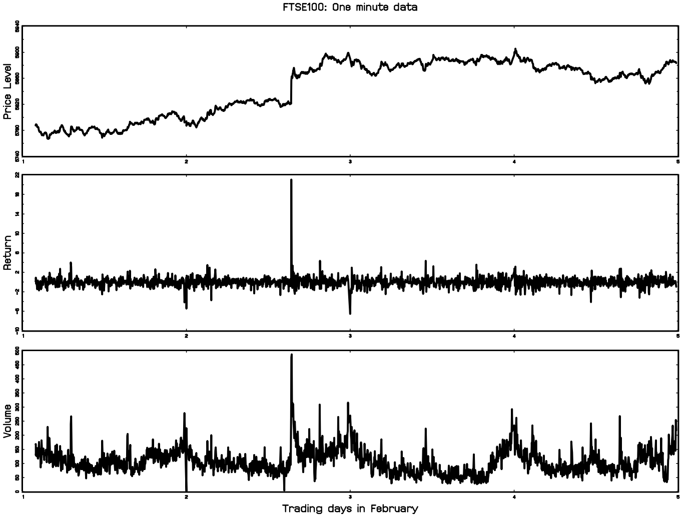
Average over splits



Some evidence that announcement effects are much faster impacted into prices. A positive US non-farm payroll day in 2012



The following web site has a long list of examples of violations of EMH and explanations thereof http://www.behaviouralfinance.net/. Here are some case studies

Example

Massively Confused Investors Making Conspicuously Ignorant Choices. Rashes (JF, 2001). He examines the comovement of stocks with similar ticker symbols MCI (large telecom, Nasdaq) and MCIC (a closed end mutual fund, NYSE). Finds a significant correlation between returns, volume, and volatility at short frequencies. New information about MCI affects prices of MCIC and vice versa. Deviations from "fundamental value" tend to be reversed within several days, although there is some evidence that the return comovement persists for longer horizons. Arbitrageurs appear to be limited in their ability to eliminate these deviations from fundamentals.

Example

CUBA, Thaler (2016). It had around 70 percent of its holdings in US stocks with the rest in foreign stocks, but absolutely no exposure to Cuban securities, since it has been illegal for any US company to do business in Cuba since 1960. For the first few months of 2014 the share price was trading in the normal 10–15 percent discount range of the Net Asset Value (the value of the shares it itself held). Then on December 18, 2014, President Obama announced his intention to relax the United States' diplomatic relations with Cuba. The price of CUBA shares jumped to a 70 percent premium Although the value of the assets in the fund remained stable, the substantial premium lasted for several months, finally disappearing about a year later.

Example

More recently, a number of firms with names overlapping with Bitcoin but with no direct connection have experienced substantial price appreciation.

Empirical Evidence on EMH

- Predictability of stock returns. Any test of weak form EMH must assume an equilibrium asset pricing model that defines 'normal' security returns against which investor returns are measured. If we reject the hypothesis that investors can't achieve superior risk-adjusted returns, we don't know if markets are inefficient or if the underlying model is misspecified. Therefore, can never reject EMH
- Excess volatility of stock prices, see below
- Event studies

Anomaly Characteristics

- They are ‘small’. Small \$ (e.g., MCI Jr. vs. MCI)
- Not scalable, e.g., illiquid
- Statistically suspect. Standard errors often based on iid. Even worse Data mining issues (**White’s Reality Check**)
- Fleeting, dont last long. E.g., the small stock premium, January effect, Monday effect. Heisenberg Principle of Finance/Goodhart’s Law (about policy instruments). Observing an anomaly brings about its extinction.
- Not realizable profit opportunities. Transaction costs: commissions and Bid/Ask spreads. Information costs, e.g., complex mortgage instruments

The degree of efficiency might be the relevant point for discussion. Comparison of inefficiency across markets or stocks or time.

Market Microstructure

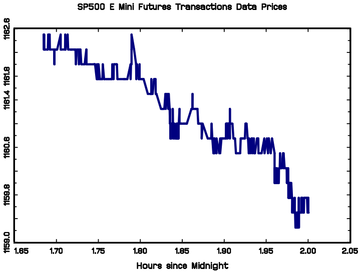
- This is concerned with how prices are actually determined in markets taking account of the fine structure of the market and how that relates to our economic models which often abstract from this. The quantum mechanics of economics.
- Several issues (stale prices, price discreteness, bid-ask bounce) generate negative autocorrelation in observed returns
- What determines the bid-ask spread? What determines the liquidity of a market?

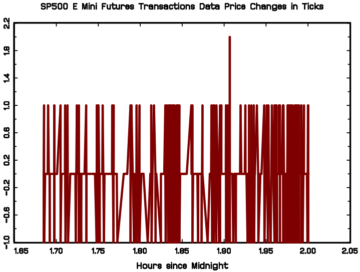
Stale Prices

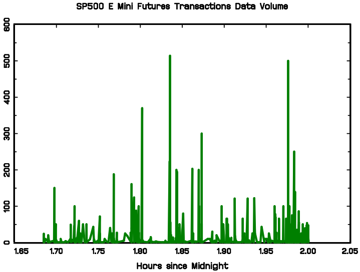
- We only observe transaction prices when there has been a trade. This means that the price may not be an accurate indication of the current value of the asset.
- In practice, stocks trade with different frequency, from Apple at one end (many times a millisecond) to "penny stocks" that may only trade once a week. Many empirical questions are concerned with the cross-section of returns, and nonsynchronous trading is a big problem in high frequency data.
- Model of infrequent trading predicts negative autocorrelation in individual stocks and positive autocorrelation in indexes consistent with empirical evidence.

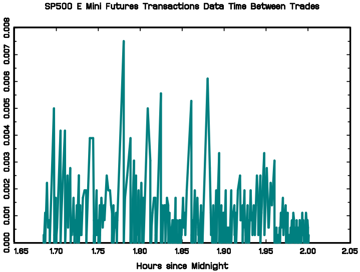
Discreteness

- Quantity and Prices are discrete (minimum price increment, minimum quantity). In 1997, the minimum price increment or **tick size** in the US was 1/8th of a dollar. Now, for any stock over $1 in price level the tick size is currently one cent (although there are exceptions - Berkshire Hathaway A priced at $134,060.00 only takes $1 moves).
- US is currently debating "subpenny pricing" whereby tick size might be reduced to 0.1 of a cent. In FX, tick size can be 0.0001 or smaller.
- In UK (and most countries except USA), tick size varies across stocks in bands according to the price level and market capitalization (generally speaking more liquid stocks have small tick sizes). Tick size has been subject to regulatory debate.
- Discrete prices can imply negative autocorrelation in observed returns when the true underlying prices are continuous


SP500 E Mini Futures Transactions Data Prices
1159.0
1159.8
1160.6
1161.4
1161.8
1162.6
1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00 2.05
Hours since Midnight


SP500 E Mini Futures Transactions Data Price Changes in Ticks
−1.0
−0.6
−0.2
0.2
0.6
1.0
1.4
1.8
2.2
1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00 2.05
Hours since Midnight


SP500 E Mini Futures Transactions Data Volume
0
100
200
300
400
500
600
1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00 2.05
Hours since Midnight


SP500 E Mini Futures Transactions Data Time Between Trades
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00 2.05
Hours since Midnight

Bid, Ask and Transaction Prices

Dealer market: Dealer/market maker quote bid and ask prices either as needed or displayed publicly (think airport currency vendor). Take it or leave it. Knows the flow of orders. In some case is a monopolist and has unique access to the order flow information; in other cases competitive dealers.

Electronic Order Book Anyone can enter buy or sell orders. Transparent display of demand and supply.

The theoretical literature is mostly concerned with a stylized version of the dealer market where:

- Market participants can be sorted into outsiders (everyone except the market maker) and insiders (the market maker).
- The market maker sets the bid ask spread. He hopes to earn the spread, i.e., to buy at the bid and sell at the ask.
- If she can make these two trades at exactly the same time, this is a money printing machine. But she cant (for reasons discussed below) and it is generally a risky business as we see.

- Roll model: What is the consequence of a (fixed) Bid-ask spread for observed price changes? Negative autocorrelation
- What determines the bid-ask spread?
    - Inventory models. Orders arrive randomly and not matched. The bid ask spread compensates the market maker for the risk of ruin through inventory explosion
    - Information models. The dealer does not know the true value of the stock, whereas a subset of traders, informed traders, do know the true value. The bid-ask spread is needed to compensate the dealer from losses incurred by trading with informed traders.
    - Combinations allow both adverse selection and inventory costs.

Strategic trade models

- How should an informed trader trade? How to measure liquidity of market?
- Kyle model (1985). Single informed trader who knows their own impact on prices and trades strategically.
- Value of security $v \sim N(\mu_v, \sigma_v^2)$. Informed trader knows v and submits demand $x(v)$ to maximize his expected profit.
- Noise traders submit order flow $u \sim N(0, \sigma_u^2)$
- Risk neutral Market maker observes total demand $y = x + u$ and then sets a price $p(y)$ to make zero profits in expectation
- Can show that in equilibirum

$$p(y) = \mu_v + \frac{1}{2}\sqrt{\frac{\sigma_v^2}{\sigma_u^2}}y \quad ; \quad x(v) = -\mu_v\sqrt{\frac{\sigma_u^2}{\sigma_v^2}} + \sqrt{\frac{\sigma_u^2}{\sigma_v^2}}v$$

Definition

Kyle's Lambda

$$\lambda = \frac{1}{2}\sqrt{\frac{\sigma_v^2}{\sigma_u^2}}$$

- This is the amount that the market maker raises the price when the total order flow y goes up by 1 unit ($\lambda = dp/dy$). Hence, the amount of order flow necessary to raise the price by \$1 equals $1/\lambda$, which is a measure of the "depth" of the market or market "liquidity."
- The higher is the proportion of noise trading to the value of insider information, the deeper or more liquid is the market. Intuitively, the more noise traders relative to the value of insider information, the less the market maker needs to adjust the price in response to a given order, since the likelihood of the order being that of a noise trader, rather than an insider, is greater.

There has been recent development of electronic trading replacing human to human trading. The development of **Algorithmic Trading** strategies and **High Frequency Trading (HFT)**. This has coincided with reduced costs of trading and communication, information availability, online brokerage (lower commissions), etc. **Reg NMS** (National Market System) in US; **MiFID** (Market in Financial Instruments Directive) in Europe permitted and encouraged the introduction of new electronic venues for trading equities. In USA, new exchanges were created such as Direct Edge, BATS etc. In UK, likewise with Chi-X.

Dark pools and **Broker Crossing Networks**. These are electronic venues with no **pre trade transparency**, i.e., the order book is not visible to participant. Price is determined usually by the midpoint of the prevailing bid and ask prices on some reference **lit exchange**. The purpose of these venues is to facilitate trading of large blocks of securities in relative secrecy thereby avoiding price impact.

There are a number of different types of orders that traders can use in electronic markets. A **Market order** is a Buy (bid) or sell (ask) order that is to be executed immediately at the current market price. There are also price-contingent orders such as **Limit orders** - these are executed only above or below a stipulated price limit. Bid - buy at or below a stated price. Ask - sell at or above a stated price. Limit orders may not execute if they are not competitive and/or the market moves away. **Stop orders** - not executed until the market price reaches a stipulated limit. Stop-buy order - buy at or above a stated price. Stop-loss order - sell at or below a stated price. **Iceberg orders** are limit orders with only a fraction of the order visible to other participants. If part of the order is executed, some of the remaining quantity may become visible. **Pegged orders** - price driven by a reference price such as the midpoint of the bid-ask spread on some other trading venue. Orders may also limited by time. Day orders expire at the end of the trading day. Open orders (good-till-cancelled, fill-or-kill) - remain in force for up to 6 months unless cancelled.

The limit order book

Bid		Ask	
Price	Quantity	Price	Quantity
15.71	2000	15.72	7000
15.70	4500	15.73	3000
15.69	5000	15.74	4000
15.68	10000	15.75	2000
15.67	15000	15.76	12000

At any one time, this is available to (some) participants.

Market orders or aggressive limit orders (that cross the spread) will execute against "the book".

Market buy order for 15000: 7000@15.72, 3000@15.73, 4000@15.74@, 1000@15.75 (walking down the book). Volume weighted average price (VWAP)

$$VWAP = \frac{7}{15} \times 15.72 + \frac{3}{15} \times 15.73 + \frac{4}{15} \times 15.74 + \frac{1}{15} \times 15.75 = 15.729$$

New order book. Bid ask spread is 0.04.

Bid		Ask	
Price	Quantity	Price	Quantity
15.71	2000	15.75	1000
15.70	4500	15.76	12000
15.69	5000		
15.68	10000		
15.67	15000		

Until replenished by new limit orders

This is **market impact**

If the market buy order were for quantity 2000, then the spread would not change. The order executes by Price Time Priority - the first order at the price gets executed first and so on

Real time limit order book: http://www.batstrading.co.uk/

Modern Markets are Very Fast

Speed has always mattered in financial markets
Nathan Mayer Rothschild profiting from news of British/Prussian victory at Waterloo in 1812

- Rothschild had news about the victory ahead of govt (fantasy version has by pigeons, others say by personal couriers)
- Two versions of how he made money
  - Just bought bonds on early news of victory
  - Spoofed the market by publicly selling bonds and having his agents buy them and keep on buying... In poker this is called bluffing, but in financial markets it could now get you 380 years in prison in the USA

System latency of LSE following Moores law or even more

System	Implementation	Latency 10^{-6}	Developments
SETS	1986-2000	600000	Decim
SETS1	Nov 2001	250000	Algos/routers
SETS2	Jan 2003	100000	
SETS3	Oct 2005	55000	Reg NMS/BATs
TradElect	June 18, 2007	15000	MIFID1
TradElect 2	October 31, 2007	11000	Reg NMS
TradElect 3	September 1, 2008	6000	
TradElect 4	May 2, 2009	5000	
TradElect 4.1	July 20, 2009	3700	
TradElect 5	March 20, 2010	3000	Flash Crash
Millenium	February 14, 2011	113	

Fibre optic cable, Microwaves. Speed of light 299,792.458 km/sec. From New York to London 5,567km

- Posting limit orders (supplying liquidity) gives options to trade to other traders. Provides a service to other traders. Needs to be compensated.
- The value of those options can be calculated from Black and Scholes (1973, JPE) call option price

$$C(S, K, \tau, r_f, \sigma) = S \cdot \Phi(d_+) - K \cdot e^{-r_f \cdot \tau} \cdot \Phi(d_-)$$

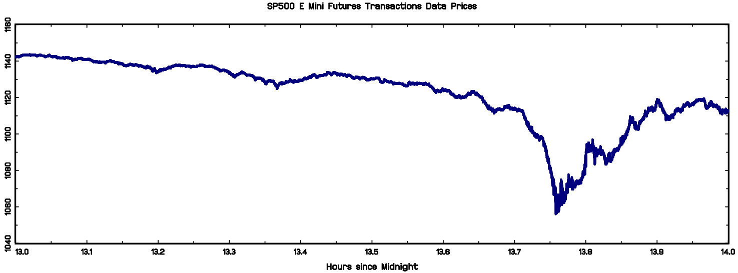
$$d_\pm = \frac{\log \frac{S}{K} + \left(r_f \pm \frac{\sigma^2}{2}\right) \cdot \tau}{\sigma \cdot \sqrt{\tau}}$$

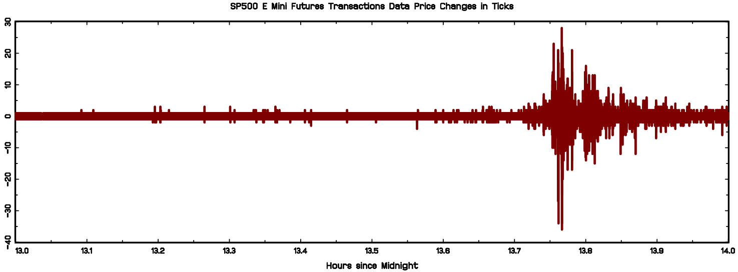
and Φ is the standard normal cdf. At the money, $S = K$, as $\tau \to 0$

$$C(\overbrace{S}^{\text{spot}}, \overbrace{K}^{\text{strike}}, \overbrace{\tau}^{\text{time to maturity}}, \overbrace{r_f}^{\text{interest}}, \overbrace{\sigma}^{\text{volatility}}) = \frac{1}{\sqrt{2\pi}} S \cdot \sigma\sqrt{\tau} + O(\tau).$$

- There is a positive albeit small value in an order that only sits for small time.
- If there are many orders, then value =large$\times$*small*
- Cost of posting orders increases with volatility and the length of time

Flash Crash in the US stock market on May 6th, 2010. Partly blamed on High Frequency (Computer) Trading (HFT) and Algorithmic (Computer) Trading (AT).





Some other recent disasters in US and other markets

Facebook IPO (Faceplant) May 18, 2012

Better Alternative Trading Systems (BATS) IPO $15.5 open traded down to a $0.01 in 1.4 seconds

Google (Pending Larry) mistaken early earnings announcement 2012, 10% in 8 minutes

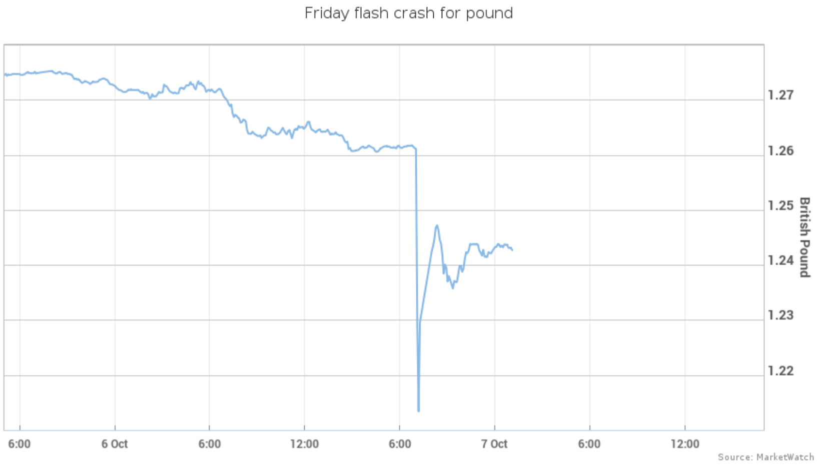
#Knightmare on Wall street. Knight Capital. US market maker/HFT. Listed on NYSE. August 1, 2012 trading error caused widespread disruption on NYSE. They lost $450m in a few minutes. Bought out by Getco.

Hash Crash, April 23rd, 2013 (hack of Reuters twitter-> "bomb at White house")

Not just equities: Treasury Flash crash/rally, October 15th, 2014, Swiss Franc depegging, January 15, 2015

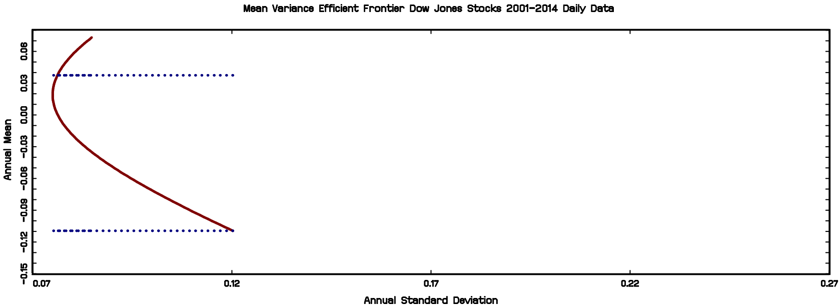
Swiss Franc depegging

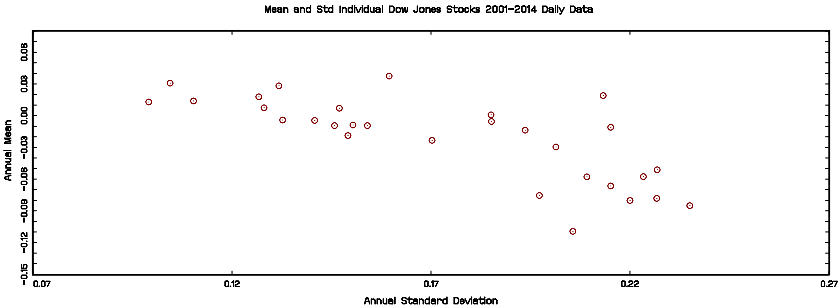
Sterling Flash Crash 20161007



Risk Return Tradeoff and Diversification: Portfolio Choice

- Individual has to decide how to invest his/her money. He cares about the **return** he gets on his investment and the **risk** that he may lose his investment. If he is risk averse, he dislikes risk.
- Many assets to choose from. General principal of **diversification**, by dividing investment into many different assets one can reduce overall risk.
- Principal well established before the mathematical treatment.
  - King Solomon in the Old Testament bible (Ecclesiastes 11:2)"*Divide your portion to seven, or even to eight, for you do not know what misfortune may occur on the earth*."
  - English saying: "*Dont put all your eggs into one basket*"
- How to find the best trade-off between risk an return to suit your own preferences. Maximize return subject to risk constraint or minimize risk subject to return constraint.


Mean Variance Efficient Frontier Dow Jones Stocks 2001-2014 Daily Data
Annual Mean
Annual Standard Deviation
0.06
0.03
0.00
-0.03
-0.06
-0.09
-0.12
-0.15
0.07
0.12
0.17
0.22
0.27


Mean and Std Individual Dow Jones Stocks 2001-2014 Daily Data
Annual Mean
Annual Standard Deviation
0.06
0.03
0.00
-0.03
-0.06
-0.09
-0.12
-0.15
0.07
0.12
0.17
0.22
0.27

The **Sharpe Ratio** gives a simple risk adjusted rate of return that is easy to understand and analyze. It is widely used to measure the performance of an investment relative to the benchmark provided by the risk free rate.

Definition

The Sharpe Ratio of asset with return R when the riskless asset return is R_f is

$$S = \frac{E(R) - R_f}{\sqrt{\mathrm{var}(R)}}.$$

Example

Consider a casino roulette wheel with 37 slots, 18 red, 18 black, and 1 void, and suppose that you bet on black. We can describe the gain per play of the Casino through the random variable X, where

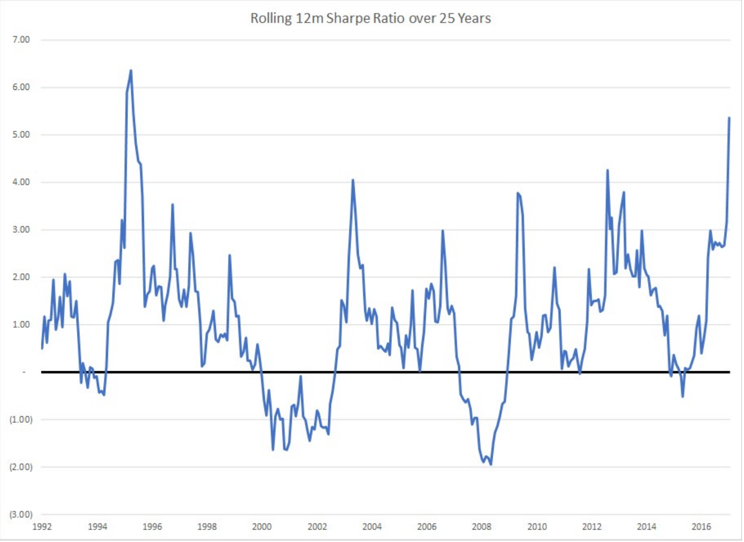
$$X = \begin{cases} 1 & 19/37 \\ -1 & 18/37. \end{cases}$$

The expected rate of return for the Casino is $1/37$. The variance per play is

$$E(X^2) - E^2(X) = 1 - \frac{1}{37^2} \simeq 1,$$

which gives a Sharpe Ratio per play of around 0.027 (we may assume $R_f = 0$ here). Compare this with stock returns. For the S&P500 from 1950-2014, the daily Sharpe Ratio is around 0.035, while the two day ratio is 0.049, and the three day ratio is 0.061.

S&P500 Sharpe Ratio



Static Asset Pricing: Capital Asset Pricing Model and Arbitrage Pricing Theory

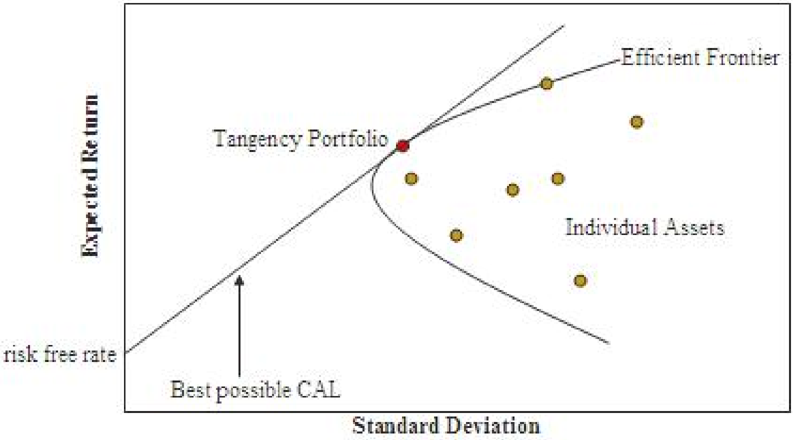
- Sharpe-Lintner (1964) version with a riskless asset (borrowing or lending).

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$

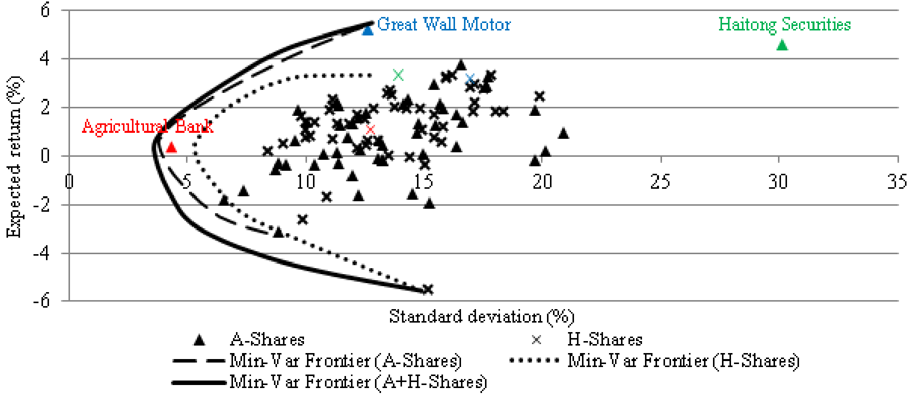
- Idiosyncratic risk versus systematic risk. By diversifying, can eliminate the idiosyncratic risk without cost, therefore should only be compensated for systematic risk, the risk that cannot be diversified.
- Expected return on asset i is proportional to the stock's beta

$$\beta_i = \frac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)}$$

- Risk/return trade-off - more risk, more return. The β_i is the relevant measure of riskiness of stock i not $\mathrm{var}(R_i)$


Efficient Frontier
Tangency Portfolio
Expected Return
Individual Assets
risk free rate
Best possible CAL
Standard Deviation

Chinese Data (Cambridge undergrad Rose Ng did this work in her thesis testing risk/return in Shanghai/HK). Two markets for same stocks.



She provides tests of pricing relationships.

Empirical Evidence in the Literature

Many tests and many rejections of the CAPM!!

- Size Effect. Market capitalization
  - Firms with a low market capitalization seem to earn positive abnormal returns ($\alpha > 0$), while large firms earn negative abnormal returns ($\alpha < 0$)
- Value effect. Dividend to price ratio (D/P) and book to market ratio (B/M).
  - Value firms (low value metrics relative to market value) have $\alpha > 0$ while growth stocks (high value metrics relative to market value) have $\alpha < 0$
- Momentum effect.
  - Winner portfolios outperform loser portfolios over medium term.

Active vs. Passive Portfolio Management

- **Active portfolio management**: attempts to achieve superior returns α through security selection and market timing in violation of CAPM and EMH
  - Security selection = picking misspriced individual securities, trying to buy low and sell high or short-sell high and buy back low
  - Market timing = trying to enter the market at troughs and leave at peaks
- Some issues with active management: Conflicts of interest between owners and managers
- **Passive portfolio management**: tracking a predefined index of securities with no security analysis whatsoever, just choose β (smart beta). Index funds, ETF's. No attempt to beat the market, in line with EMH, which says this is not possible. Much cheaper than active management since no costs of information acquisition and analysis, lower transaction costs (less frequent trading), also generally greater risk diversification (the only free lunch around).

- There are is a lot of work evaluating the performance of active managers. If an active manager overseeing a £5 billion portfolio could increase the annual return by 0.1%, her services would be worth up to £5 million. Should you invest with her?
- In evaluating this, consider the role of luck:
  - Imagine 10,000 managers whose strategy is to park all assets in an index fund but at the end of every year use a quarter of it to make (independently) a single bet on red or black in a casino. After 10 years, many of them no longer keep their jobs but several survivors have been very successful ($(1/2)^{10} \approx 1/1000$).
  - The **infinite monkey theorem** says that if one had an infinite number of monkeys randomly tapping on a keyboard, with probability one, one of them will produce the complete works of Shakespeare.

Criticisms of Mean-Variance analysis and the CAPM

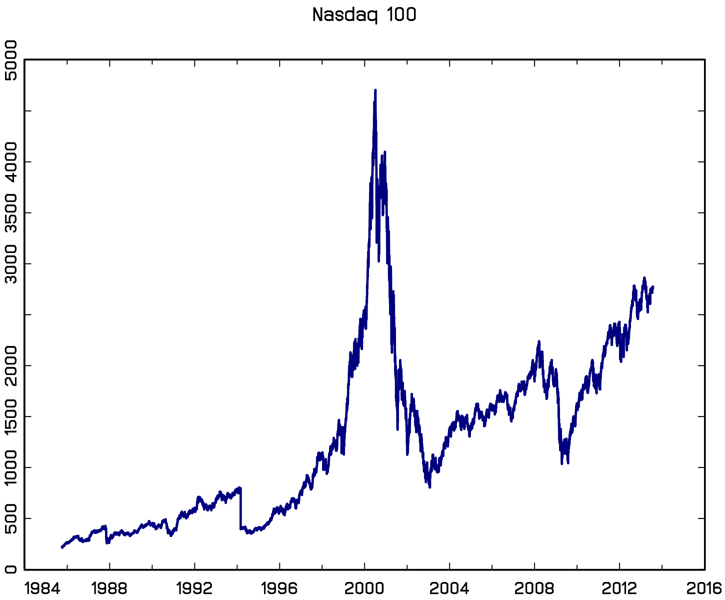
- Roll critique. Cant observe the market portfolio. So rejections of CAPM are not valid
- Normality (or an Elliptic distribution) is crucial to the derivation of the CAPM. The Normal distribution is statistically strongly rejected in the data.
- Furthermore, the CAPM has only negligible explanatory power.
- Ex ante versus ex post betas. Conditional CAPM. Time varying risk premia. For example recession indicators. Will discuss later.

Dynamic Asset Pricing: Fundamentals versus Bubbles

- Are stock prices (We now focus on prices rather than returns) driven by fundamental values or rational expectations of what those fundamental values will be in the future?
  - Efficient markets hypothesis, CAPM, etc
- Or are stock prices driven by "animal spirits", fads, bubbles, and irrational exuberance?
  - Behavioural finance, Kahneman and Tversky. Shiller (2000). Irrational Exuberance. Thaler (2016).
- Sometimes one and sometimes other.

Bubbles and Crashes

- Many examples of "bubbles" in financial history:
  - tulips in Amsterdam in 17th century
  - south sea bubble 18th century
  - 1920s? 1987? 2000?. There is less agreement on these.
  - Bitcoin?
- Typically think of bubbles as a pervasive market wide phenomenon with rapid increases ultimately followed by a crash.
- Is there a rational explanation for them, or are they only explicable due to animal spirits and irrationality?


Nasdaq 100
0
500
1000
1500
2000
3000
3500
4000
5000
1984
1988
1992
1996
2000
2004
2008
2012
2016

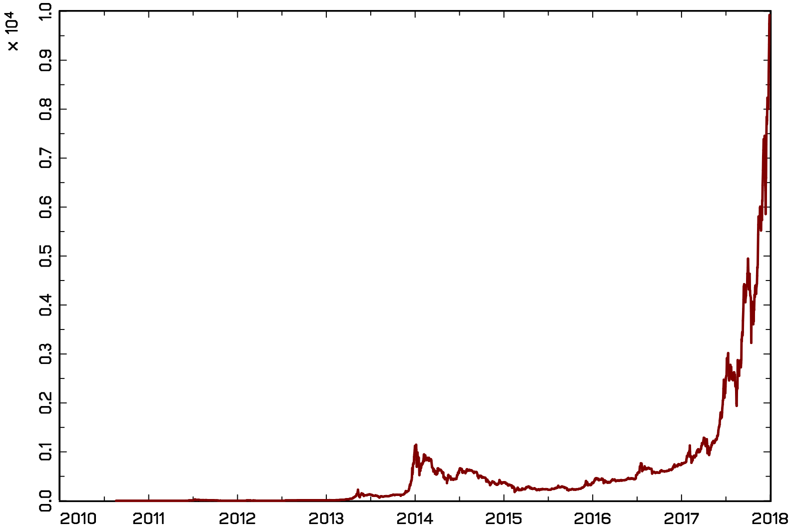


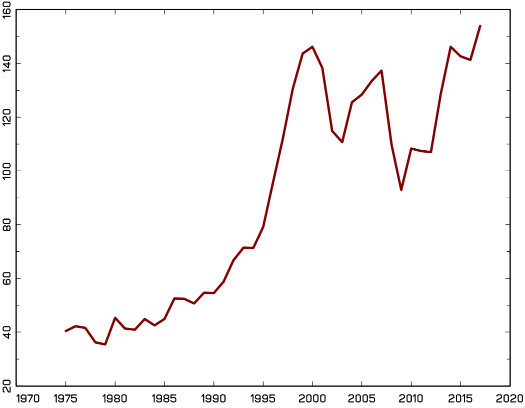
Figure: Bitcoin price level from 2010-2017

Shiller (2000, Irrational Exuberance) Table 6.1

Largest Recent One-Year Real Stock Price Index Increases

	Percentage	One year period	Subsequent ΔPrice
Philippines	683.4	Dec 1985-Dec 1986	28.4
Taiwan	400.1	Oct 1986-Oct 1987	65.7
Venezuela	384.6	Jan 1990-Jan 1991	33.1
Peru	360.9	Aug 1992-Aug 1993	15.8
Columbia	271.3	Jan 1991-Jan 1992	-19.9
Jamaica	224.5	April 1992-April 1993	-59.2
Chile	199.8	Jan 1979-Jan 1980	38.9
Italy	166.4	May 1985-May 1986	-15.7
Jamaica	163.4	Aug 1985-Aug 1986	8.7
Thailand	161.9	Oct 1986-Oct 1987	-2.6
India	155.5	April 1991-April 1992	-50.3
Italy	147.3	April 1980-April 1981	-32.1
Austria	145.4	Feb 1989-Feb 1990	-19.8
Finland	128.3	Sept 1992-Sept 1993	46.3
Denmark	122.9	April 1971-April 1972	-12.4

- Warren Buffet: Total market cap to GDP (%). Shows the US might be in a bubble



- But the measure is flawed:
  - GDP is a flow, market valuation is a stock. We see below how these may relate
  - Companies that make up the US market earn a substantial amount of profit overseas.
  - Corporate margins and thus profits as a percent of GDP fluctuate over time.
  - The proportion of public companies to private companies also fluctuates over time and impacts the total market cap calculation.

After the bubble comes the crash...

- Famous examples of stock market crashes include: the US markets on October 24th 1929 and October 19th 1987. May 6th, 2010 Flash Crash (no prior bubble)

- Each of these market events have been the subject of a lot of research and newspaper coverage as well as government reports. Typically these reports do not identify a single causal explanation but several factors that played a role. So even in these well known extreme cases it is hard to pin specific blame.

- Scheinkman (2014, Arrow Lecture). Three stylized facts about bubbles:
  - Asset price bubbles coincide with increases in trading volume
  - Asset price bubbles often coincide with financial or technological innovation
  - Asset price implosions seems to coincide with increases in assets's supply
- There are authors who dispute the bubble explanations.
  - Garber (1990) proposes market fundamental explanations for the three famous historical bubbles.
  - Pastor and Veronesi (2006) argue that the turn of the 20th century tech bubble was at least partly explained by an increase in the uncertainty about average future profitability in the late 1990s.

What are fundamentals?

- Macroeconomic news affecting all firms, in some sectors/countries exchange rates are important
- Weather can be important for retail and agriculture,
- Earnings, Dividends of individual firms

Volatility

- The extent to which prices go up and down
  - Extreme volatility bad for market confidence especially retail investors
  - Volatility can reflect upside and downside price movements associated with innovation, creative destruction of value in old industries

Volatility Modelling

- Risk/Volatility measurement is central to finance
  - Asset pricing. Conditional CAPM

$$E_{t-1}r_{i,t} - r_f = \beta_{i,t}\lambda_t$$

$$\beta_{i,t} = \frac{\mathrm{cov}_{t-1}(r_{i,t}, r_{m,t})}{\mathrm{var}_{t-1}(r_{m,t})}$$

  - Risk Management/Value at Risk

$$VaR_t(\alpha) = \mu + \overbrace{\sigma_t}^{\text{volatility forecast}} \times \overbrace{q_\alpha}^{\text{quantile of innovation}}$$

  - Portfolio Allocation

$$\max_{w \in S_d} w^\intercal E_{t-1}(\mathbf{r}_t) \text{ s.t. } w^\intercal \mathrm{var}_{t-1}(\mathbf{r}_t) w = \sigma^2$$

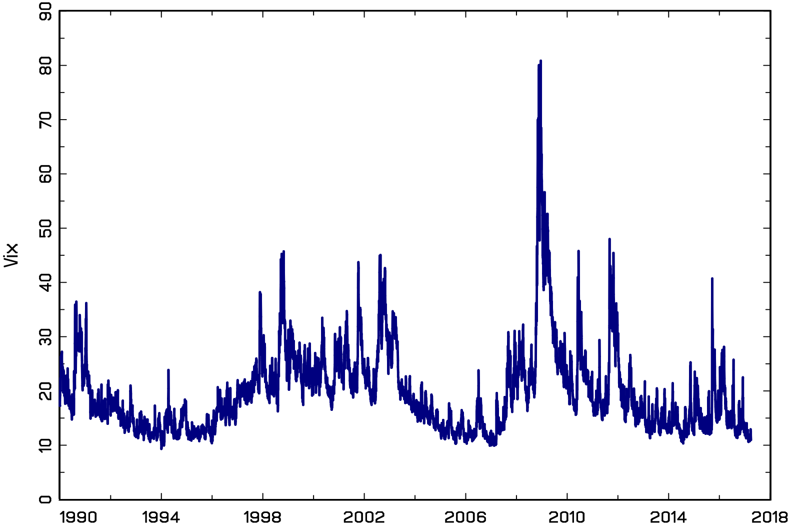
  - Measuring market quality - highly volatile markets discourage participation

- Different ways of measuring volatility, ex post and ex ante
  - Implied Volatility from Option Prices (VIX)
  - Realized volatility from high frequency data and realized range from daily high and low prices

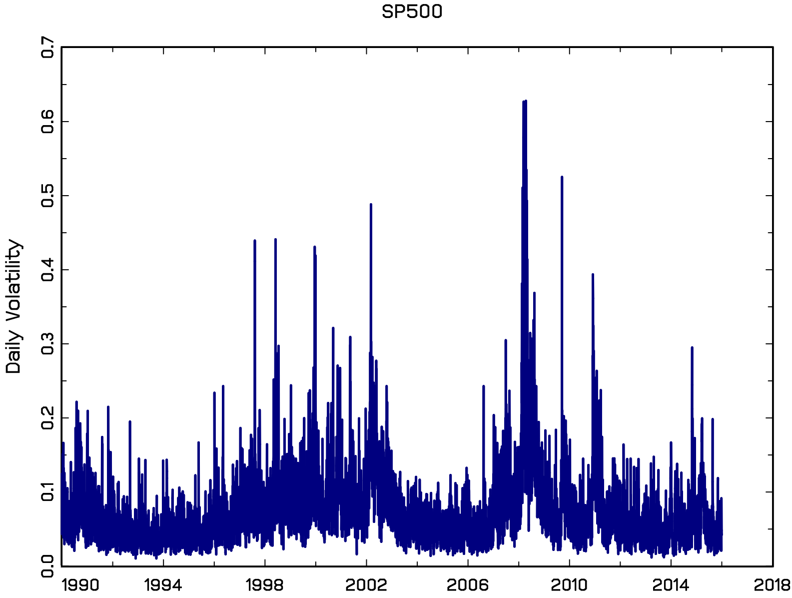
$$vol = \frac{HighPrice - LowPrice}{LowPrice}$$

  - Dynamic time series volatility models

The VIX


Vix
0
10
20
30
40
50
60
70
80
90
1990
1994
1998
2002
2006
2010
2014
2018

Realized range



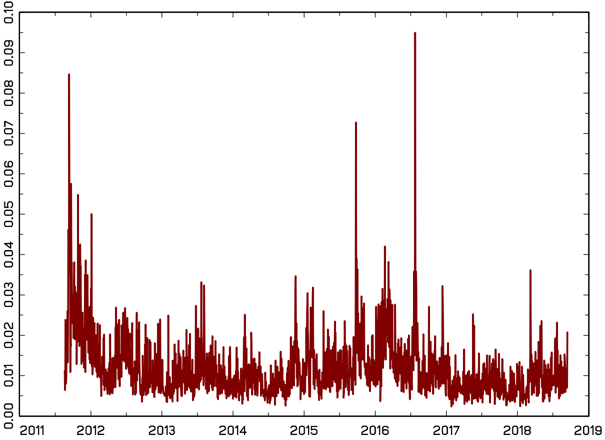


Figure: FTSE100 Volatility: Median value 0.0098400810

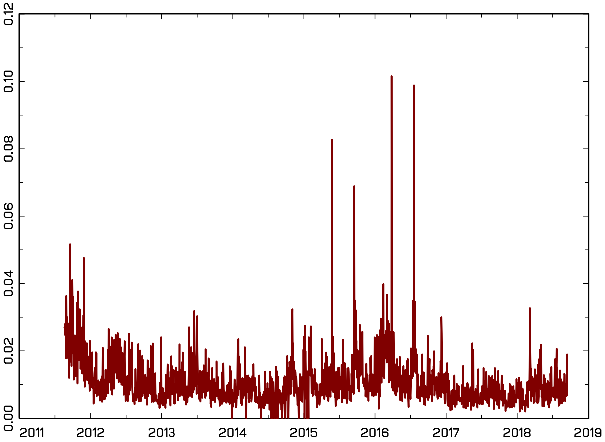


Figure: FTSE Allshare Volatility: Median value 0.0088861958

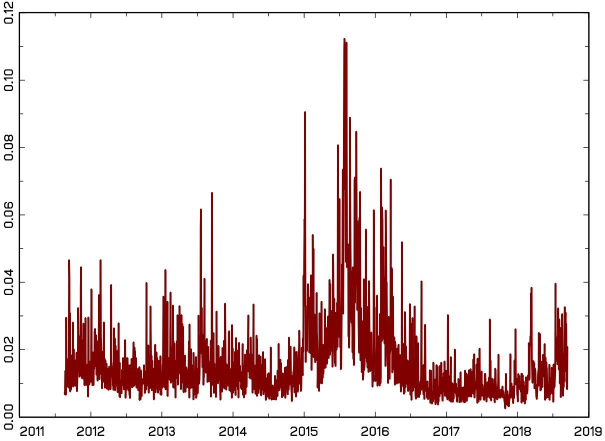


Figure: Shanghai Volatility: Median Value 0.012213816

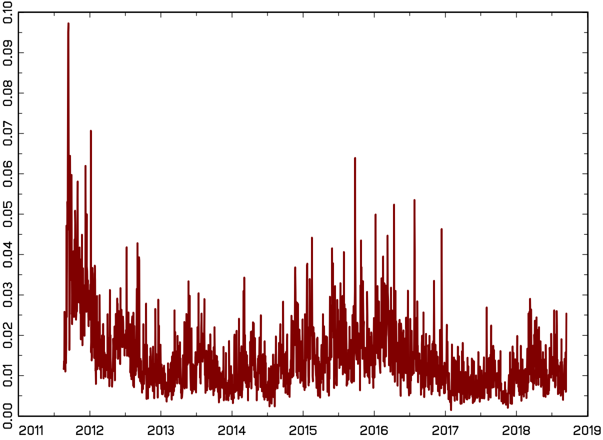


Figure: DAX Volatility: Median value 0.012323353

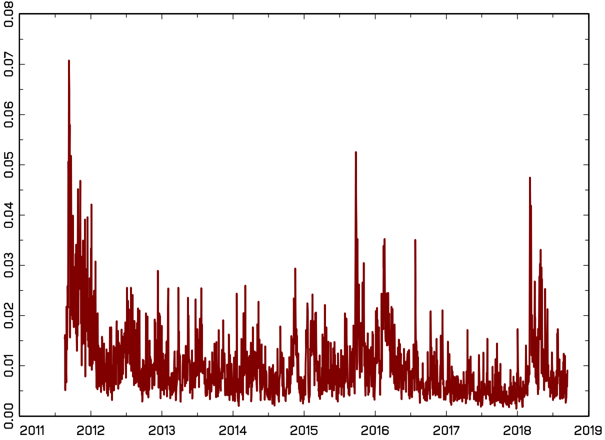
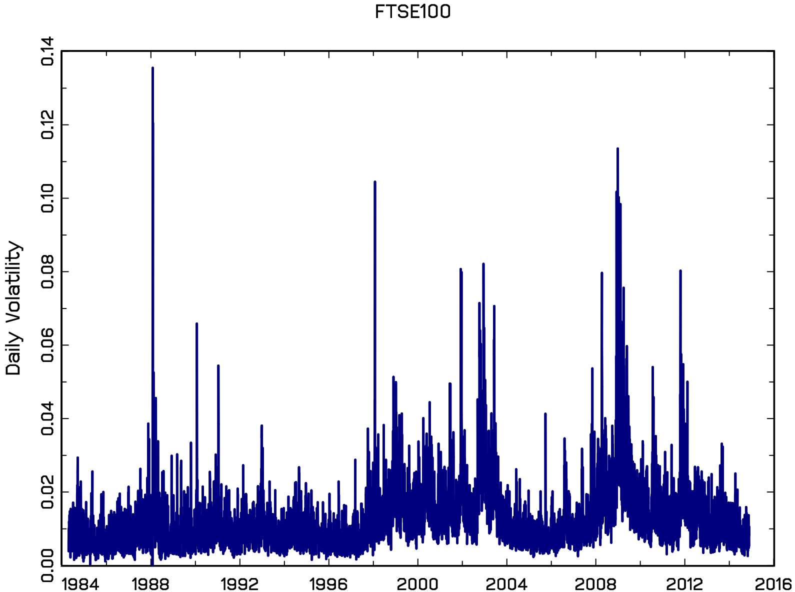


Figure: S&P500 Volatility: Median value 0.0080660960

Further back in time



FTSE100 Top 20 Most Volatile days since 1984 (- means $P_C < P_O$, + means $P_C > P_O$)

Date	Volatility
19871020	0.131-
19871022	0.115-
20081010	0.112-
19971028	0.096-
20081024	0.096-
20081006	0.094-
20081008	0.094-
20080919	0.093+
20081124	0.090+
20081015	0.084-
19871019	0.081-
20020920	0.080+
20081013	0.076+
20010921	0.076-
20081029	0.075+
20110809	0.074+
20090114	0.074-
20080122	0.074+
20020715	0.071-
20081016	0.070-

S&P500 Top 20 Most Volatile days since 1960

Date	Volatility
19871019	0.257-
19871020	0.123+
20081010	0.107-
20081009	0.106-
20081113	0.104+
20081028	0.101+
20081015	0.100-
20081120	0.097-
20081013	0.094+
20080929	0.093-
19871026	0.092-
20100506	0.090-
20081201	0.089+
19620529	0.089+
19871021	0.087+
20081016	0.087+
20081006	0.085-
20081022	0.085-
20020724	0.081+
19980831	0.080-

US market more volatile than UK perhaps explained by more innovation! Both sides of Atlantic dominated by 2008 and 1987. Circuit breakers now limit the worst case or perhaps spread it out over several days

Volatility and EMH

- There is substantial evidence that volatility of asset returns varies over time in a way that can be partially predicted. Does this violate market efficiency?
- The answer is no unless a trading strategy could be designed that would use this information in the options markets to identify under- and over-valued options. If options markets are efficient, option prices should incorporate the best volatility forecasts at all points in time

Schwert (1989)

- He examines monthly US volatility computed as sum of squared daily returns for 1885-1987 and a regression model approach like GARCH for 1857-1985.
- Main findings
  - The average level of volatility is higher during (NBER dated) recessions
  - The level of volatility during the great Depression was very high
  - The effect of financial leverage on volatility is small
  - There is weak evidence that macroeconomic volatility can help to predict financial asset volatility and stronger evidence for the reverse prediction
  - The number of trading days in the month is positively related to stock volatility (Trading days per year NYSE 252, LSE 255 (but 24 Dec is half day))
  - Share trading volume growth is positively related to stock volatility

French and Roll (1986)

Volatility over weekend and holidays.

- Calendar time hypothesis: Variance is proportional to calendar time
- Trading time hypothesis: Variance is proportional only to the trading time

Typical trading day may be 8 hours long out of 24 hours (say 8-4).
Weekend, Friday close to Monday open contains 64 hours.
Suppose that hourly stock returns satisfy

$$r_t \sim \mu_h, \sigma_h^2$$

Then daily returns (open to close) satisfy

$$r_t \sim 8\mu_h, 8\sigma_h^2$$

Monday open from friday close returns satisfy

$$r_t \sim \begin{cases} 64\mu_h, 64\sigma_h^2 & \text{Calendar time} \\ 0 & \text{Trading Time} \end{cases}$$

They calculate variance as the average of squared returns over stocks and over the relevant period

$$varperhour_{monday} = \frac{1}{n_{monday}} \sum_{mondays} \frac{\left(p_{mclose} - p_{mopen}\right)^2}{8}$$

$$varperhour_{weekend} = \frac{1}{n_{weekend}} \sum_{weekends} \frac{\left(p_{mopen} - p_{fclose}\right)^2}{64}$$

They find that per hour return variance is 70 times larger during a trading hour than during a weekend hour

Is this because

1. Volatility is caused by public information which is more likely to arrive during normal business hours
2. Volatility is caused by private information which affects prices when informed investors trade
3. Volatility is caused by pricing errors that occur during trading

They find:

- There is some misspricing but most is caused by information release
- To distinguish between public and private information (explanations 1 and 2) they use the fact that in 1968, NYSE was closed every wednesday because of "paperwork crisis", but otherwise was a regular business day. Explanation 2 is their main story.

Annualized std, and idiosyncratic std

	σ	σ_ε		σ	σ_ε
Alcoa Inc.	0.2151	0.1676	JP Morgan	0.0991	0.0806
AmEx	0.1851	0.1160	Coke	0.2234	0.1591
Boeing	0.2350	0.1717	McD	0.1491	0.1367
Bank of America	0.1540	0.1191	MMM	0.1458	0.1217
Caterpillar	0.1595	0.1168	Merck	0.1972	0.1811
Cisco Systems	0.1103	0.0923	MSFT	0.2092	0.1818
Chevron	0.2151	0.1695	Pfizer	0.2057	0.1901
du Pont	0.1504	0.1251	P & Gamble	0.1045	0.0872
Walt Disney	0.1408	0.0980	AT&T	0.1327	0.1058
General Electric	0.2267	0.2007	Travelers	0.1703	0.1402
Home Depot	0.2200	0.1862	United Health	0.2132	0.1968
HP	0.1937	0.1614	United Tech	0.1852	0.1578
IBM	0.2014	0.1686	Verizon	0.1280	0.1034
Intel	0.1318	0.0978	Wall Mart	0.1268	0.1022
Johnson[2]	0.2268	0.1880	Exxon Mobil	0.1470	0.1218

Fixed Income Modelling

- Bonds are contracts that specify fixed payments of cash at specific future dates. They are a big market: US Treasury Securities $\$21 \times 10^{12}$ outstanding in 2017, Japanese securities $\$10.46 \times 10^{12}$ outstanding in 2013.
- There are several types of bonds in the US market:
  - **Zero Coupon Bonds** (single payment at a specified future time), also known as zeros;
  - **T-bills** with original maturities of less than a year;
  - **Coupon Bonds** (Coupon bonds have coupon payments c expressed as a percentage of redemption value, which is set equal to \$100. Coupon is paid every six months (or year) until maturity at which date the holder receives \$100);
    - ⋆ **Notes** with original maturities from 1 to 10 years;
    - ⋆ **Bonds** with original maturities more than 10 years.

Definition

The present value of a coupon bond paying c in each period and 1 at the end of n periods is the present discounted value of future cash flows

$$pv = \frac{c}{(1+i)} + \frac{c}{(1+i)^2} + \ldots + \frac{1}{(1+i)^n},$$

where i is the interest rate or discount factor

Definition

The yield to maturity y on a coupon bond paying c in each period and 1 at the end of n periods is implied by the relation

$$p = pv = \frac{c}{(1+y)} + \frac{c}{(1+y)^2} + \ldots + \frac{1}{(1+y)^n},$$

where p is the price. The par bond yield curve is the graph of the yield to maturity of coupon bonds that sell at par.

The choice of period matters in the above formula. It is convenient to work with infinitesimal time periods, i.e., **continuous compounding**.

Definition

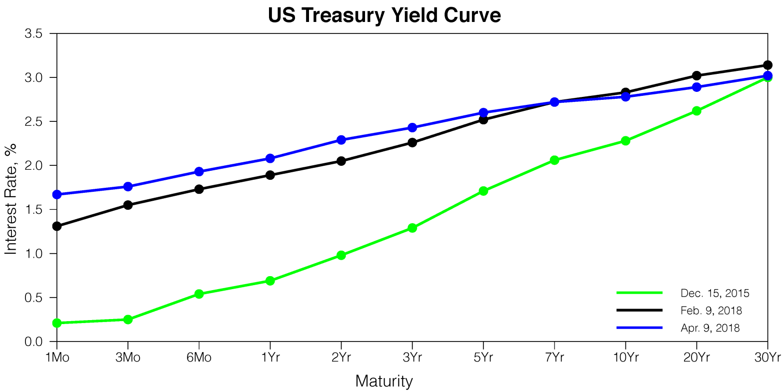
The discount function at maturity τ denoted $d(\tau)$ is the price of one dollar received at point τ in the future. The discount function and yield curve d and y are related by

$$d(\tau) = \exp(-\tau y(\tau)),$$

where $y(\tau)$ is the yield, i.e., the rate of interest on a zero-coupon payment at maturity τ. The yield curve is related to the forward rate by

$$y(\tau) = \frac{1}{\tau} \int_0^\tau f(s) ds,$$

where $f(s)$ is the forward short term rate applicable at time s. The quantities d, y, and f are equivalent given the boundary conditions $d(0) = 1$ and $d(\infty) = 0$. The discount function is monotonically decreasing.



Data via FRED * Chart created: 04/09/2018 16:57

There are several theories about how interest rates/yields are determined.

Expectations hypothesis. Todays forward rates are expectations of future one period spot rates, that is, the expected rate of return from rolling over short term bonds must be equal to the rate of return from holding the long bond to maturity.

Liquidity (Risk) Premium Hypothesis. Todays forward rates are equal to the expectations of future short rates plus a premium to make long term bonds as liquid as short term bonds. People on average are prepared to pay a premium to hedge against future macroeconomic uncertainty, and hence prefer to pay a higher interest rate on longer term bonds.

Market Segmentation Hypothesis. Different maturity sectors represent distinct markets with their own demand and supply forces. For example, regulatory or other constraints may force a need for instruments with particular maturities.

Preferred Habitat. Same as the market segmentation hypothesis except that investors will deviate from their desired maturity sector if offered a premium

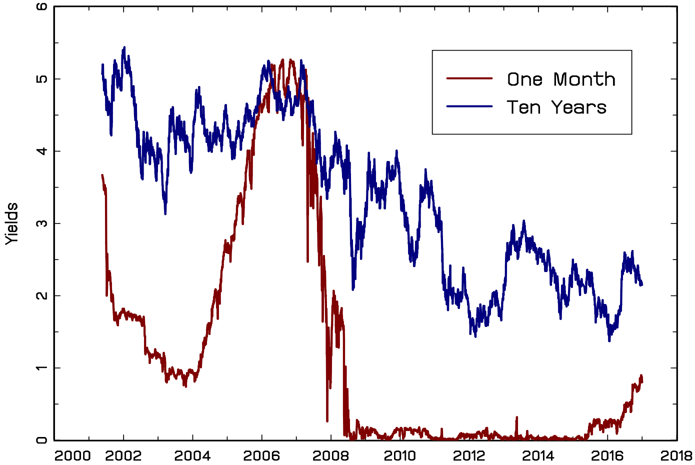


Figure: Time series of one month and ten year yields since 2000

In Figure we examine the daily one month and one year T-bill interest rate series over the period 2002-2017. The series look quite unusual, especially since 2009 since both have been close to zero.

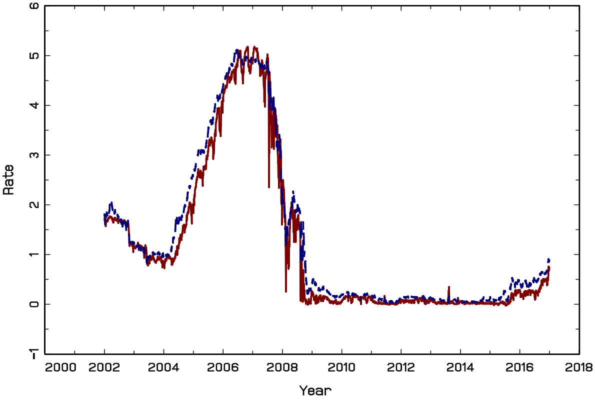


Figure: Daily Tbill Rates

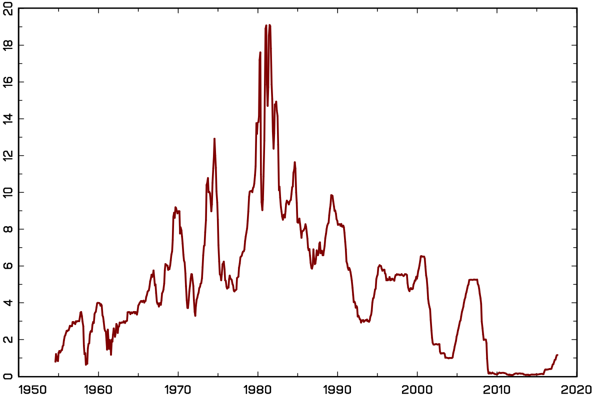
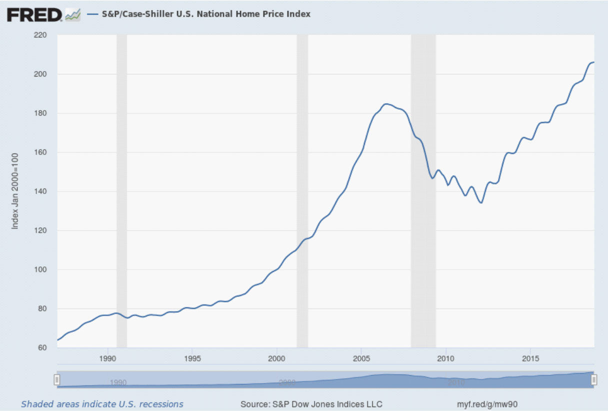


Figure: Daily Federal Funds rate annualized

Financial Crisis 2007-2009

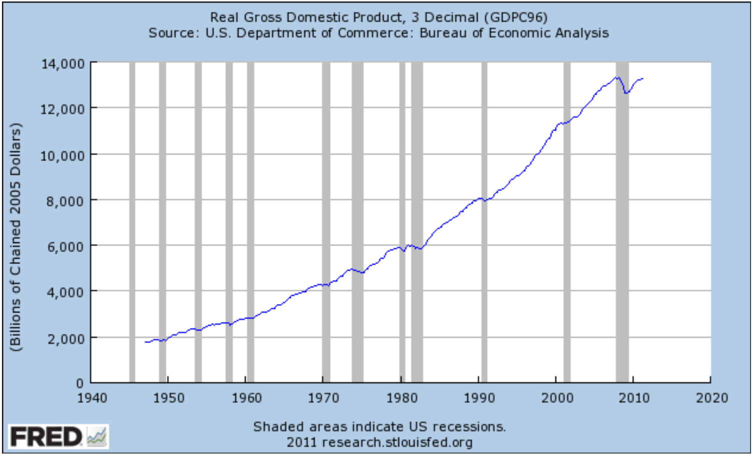
- For years prior to 2007 Many mortgages "subprime"
  - little if any equity
  - excessive payments relative to income
  - often no documentation of value or income
  - NINJA mortgages: No Income, No Job, No Assets
- Encouraged by Fannie Mae and Freddie Mac. Govt sponsored mortgage intermediaries
- Led to self-sustaining bubble in house prices 2007-2008
- Borrowers realized they were overextended, cut back on spending, borrowing
- Housing bubble stalled, then started to pop


FRED
S&P/Case-Shiller U.S. National Home Price Index
Index Jan 2000=100
220
200
180
160
140
120
100
80
60
1990
1995
2000
2005
2010
2015
Shaded areas indicate U.S. recessions
Source: S&P Dow Jones Indices LLC
myf.red/g/mw90

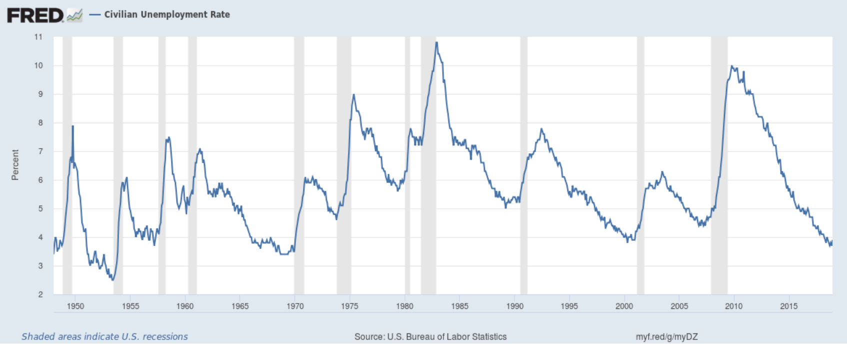
- Delinquencies and foreclosures spread Washington Mutual Savings & Loan failed Countrywide Financial failed
- Fannie and Freddie rescued by Fed in 2008 Investment banks Bear Stearns, Lehman Bros failed Bank Holding Companies in trouble
- GDP fell, Unemployment rose.

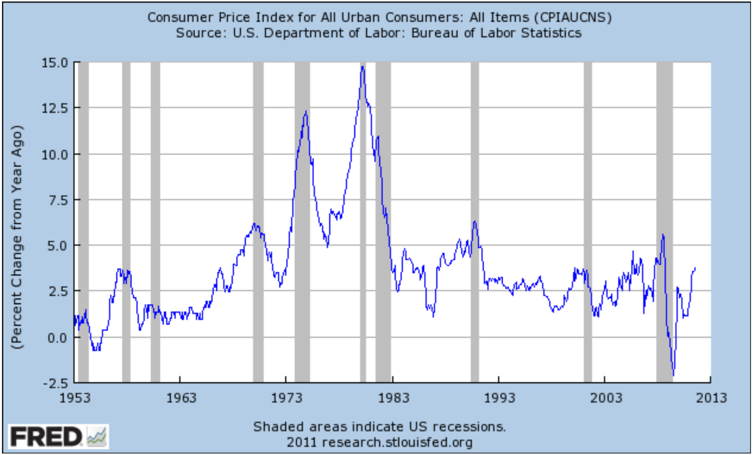
How serious compared to past recessions?

Pundits often say "worst since Great Depression of 1930s." Is this true? Or just alarmism?



The recession of 2008-9 slightly worse than 1957-58 but 1944-47 twice as bad, 1929-33 six times worse


FRED
Civilian Unemployment Rate
Percent
11
10
9
8
7
6
5
4
3
2
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Shaded areas indicate U.S. recessions
Source: U.S. Bureau of Labor Statistics
myf.red/g/myDZ


Consumer Price Index for All Urban Consumers: All Items (CPIAUCNS)
Source: U.S. Department of Labor: Bureau of Labor Statistics
15.0
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
(Percent Change from Year Ago)
1953
1963
1973
1983
1993
2003
2013
FRED
Shaded areas indicate US recessions.
2011 research.stlouisfed.org